

Modeling Mechanical and Electrical Uncertain Systems using Functions of Robust Control MATLAB Toolbox®3

Mohammed Tawfik Hussein

Electrical Engineering Department, Faculty of Engineering
Islamic University of Gaza
Gaza City, Gaza Strip- Palestine

Abstract—Uncertainty is inherent property of all real life control systems, and this is due to that there is nothing constant practically; all parameters are going to change under some environmental circumstances, therefore control engineers must not ignore this changing since it can affect the behavior and the performance of the system.

In this paper a critical research method for modeling uncertain systems is demonstrated with the utilization of built in robust control Mat-lab Toolbox®3 functions. Good results were obtained for testing the stability of interval linear time invariant systems.

Finally mechanical and electrical uncertain systems were implemented as practical example to validate the uncertainty.

Keywords—uncertainty; interval; robust stability; system response; Nyquist criteria; root bounds

I. INTRODUCTION

Robustness is of crucial importance in control-system design because real engineering systems are vulnerable to external disturbance and measurement noise and there are always differences between mathematical models used for design and the actual system. Typically, a control engineer is required to design a controller that will stabilize a plant, if it is not stable originally, and satisfy certain performance levels in the presence of disturbance signals, noise interference, unmodeled plant dynamics and plant-parameter variations.

In general, there are two categories of control systems, the open-loop systems and closed-loop systems. An open-loop system uses a controller or control actuator to obtain the design response.

A closed-loop control system uses sensors to measure the actual output to adjust the input in order to achieve desired output.

In this paper building uncertain system models using the functions of Robust Control Toolbox®3 is presented. Modeling and analyzing such systems is an important and essential step towards robust control system design. The corresponding functions of Robust Control Toolbox®3 allow to facilitate the process of building different uncertainty models and to analyze easily the properties of such models. First the description of building models of open-loop and

closed-loop linear time-invariant systems (LTI models) is introduced along with their basic properties.

Then various functions of Robust Control Toolbox®3 were used to allow creating models of systems with structured (real) uncertainties. The usage of these functions is illustrated for the simple case of a second order mass-damper-spring system and the RLC electrical circuit. It is shown how to investigate several properties of uncertain models in the time domain and frequency domain.

A. LTI Models

This section is dealing with developing and manipulating models of linear time invariant systems (LTI models) in MATLAB®.

Creation of LTI models of multivariable systems is done by the following commands:

- ss—State-space models (SS objects)
- tf—Transfer function matrices (TF objects)
- zpk—Zero-pole-gain models (ZPK objects)
- frd—Frequency response data models (FRD objects)

B. Literature Review

The problem of an interval matrices was first presented in 1966 by Ramon E. Moore, who defined an interval number to be an ordered pair of real numbers $[a, b]$, with $a \leq b$ [1]-[2].

This research is an extension and continuation to the previous publications and ongoing research of the author [3]-[7].

An interval number $[a, b]$ is defined to be the set of x such that $a \leq x \leq b$. The arithmetic operations on intervals are defined as follows:

$$[a, b] + [c, d] = [a + c, b + d]$$

$$[a, b] \times [c, d] = [\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)];$$

$$[a, b] - [c, d] = [a - d, b - c];$$

$$[a, b] \div [c, d] = [a, b] \times [1/c, 1/d]$$

The above four interval equations are programmed as Matlab functions as shown below, these are `intadd`, `intsub`, `intmul` and `intdiv`

`Intadd:`

```
function [ c ] = intadd( a,b )
%UNTITLED2 Summary of this function goes
here
% Detailed explanation goes here
c=[a(1,1)+b(1,1), a(1,2)+b(1,2)]
end
```

intsub:

```
function [ c ] = intsub( a,b )
%UNTITLED2 Summary of this function goes
here
% Detailed explanation goes here
c=[a(1,1)-b(1,2), a(1,2)-b(1,1)]
end
```

intmul:

```
function [ c ] = intmul( a,b )
%UNTITLED2 Summary of this function goes
here
% Detailed explanation goes here
a=[((a(1,1))*(b(1,1))), ((a(1,1))*(b(1,2))), ((a(1,2))*(b(1,1))), ((a(1,2))*(b(1,2)))]];
c=[min(a),max(a)];
end
```

intdiv:

```
function [ c ] = intdiv( a,b )
%UNTITLED2 Summary of this function goes
here
% Detailed explanation goes here
c=intmul([(a(1,1)), (a(1,2))], [(1/(b(1,2))), (1/(b(1,1)))]);
end
```

II. METHODOLOGY AND SIMULATION

In this research the design and evaluate the robust stability for three dynamic electrical and mechanical systems were presented.

Based on Moore famous four interval arithmetic, all possible matrices of the interval (uncertain) state matrix A of system state space model are computed, also plotting step response and bode diagram for each new matrix which result in an envelope with its upper and lower bounds, find all polynomials of the family matrix in order to compute and plot the convex hull of the system and finally plotting Nyquist and the roots bounds of the interval system.

Mat-lab 2013 software is used with some of its robust functions and commands to design and analysis the system stability and to get the convex Hull and eigenvalues bounds plots. Therefore this paper is a continuation and extension efforts of the author previous work dealing with the robust stability of an interval or uncertain system, as an efficient and helpful tool for control systems engineers [8-15]. The following three different unique engineering examples will be used to validate and demonstrate the methodology and used technique.

III. EXAMPLE 1: MASS- SPRING- DAMPER SYSTEM

The following example that is shown in figure 1 presents a mass- spring Damper as a mechanical system whose parameters are suffering from uncertainty and hence deviations from the nominal values, due to several conditions such as ageing, temperature or other disturbances.

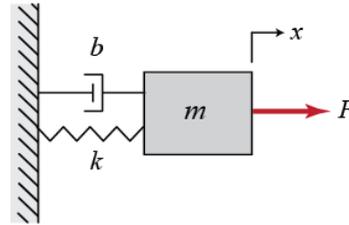


Fig. 1. Mass- Spring-Damper system

The free body diagram for this system is illustrated below in Fig. 2.

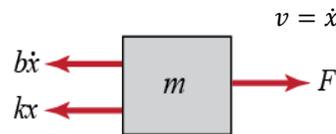


Fig. 2. Free body diagram

Applying Newton's second law by summing the forces as shown in the equation below:

$$\Sigma F_z = F(t) - b\dot{x} - kx = m\ddot{x}$$

To determine the state-space representation of the mass-spring-damper system, from the system differential equations the state space representation is derived by selecting the position and velocity as system state variables. Also system parameters are shown below in table I.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f$$

$$y = [1 \quad 0] \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$$

TABLE I. THE PHYSICAL PARAMETERS FOR MASS SPRING-DAMPER SYSTEM

M	Mass	1.0 kg
K	spring constant	1.0 N/m
B	damping constant	0.2 Ns/m
F	input force	1.0 N

With 10% variation in mass and spring constant of physical system and constant damping parameter, the interval parameters are as follows:

$$m=[0.9 \quad 1.1] \quad k=[0.9 \quad 1.1] \\ b=0.2$$

The system state interval matrix A with these

specifications is shown below:

$$\begin{bmatrix} 0 & & & 1 \\ [-11/9 & -9/11] & [-0.2/0.9 & 0.2/1.1] \end{bmatrix}$$

Using Mat-Lab, $2^2 = 4$ sub-matrices can be generated from the above interval A- matrix as shown below

$m1 = \begin{bmatrix} 0 & 1.0000 \\ -1.2222 & -0.2222 \end{bmatrix}$	$m2 = \begin{bmatrix} 0 & 1.0000 \\ -1.2222 & -0.1818 \end{bmatrix}$
$m3 = \begin{bmatrix} 0 & 1.0000 \\ -0.8182 & -0.2222 \end{bmatrix}$	$m4 = \begin{bmatrix} 0 & 1.0000 \\ -0.8182 & -0.1818 \end{bmatrix}$

And its corresponding four (4) polynomials were computed (using mat-lab) as follows:

$$\begin{aligned} po1 &= 1.0000 & 0.2222 & 1.2222 \\ po2 &= 1.0000 & 0.1818 & 1.2222 \\ po3 &= 1.0000 & 0.2222 & 0.8182 \\ po4 &= 1.0000 & 0.1818 & 0.8182 \end{aligned}$$

The analysis of open and closed step responses for the spring damper system is shown below in Fig. 3 and 4 respectively.

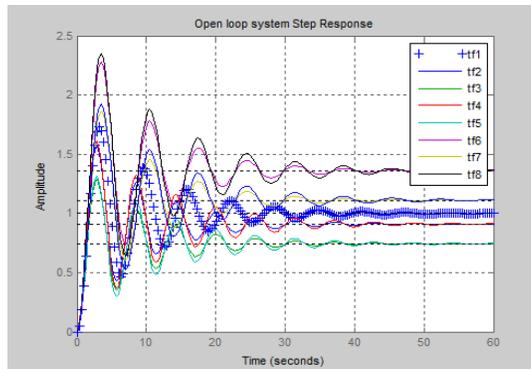


Fig. 3. Open loop system response

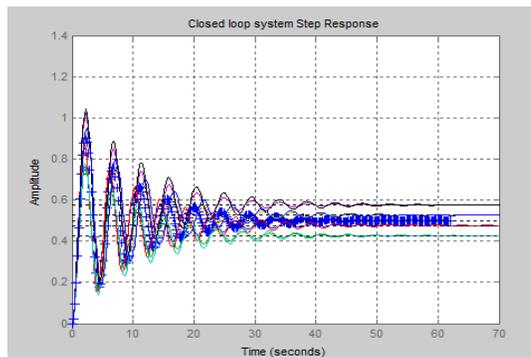


Fig. 4. Closed loop system response

Bode diagram (open loop Vs. closed loop) is shown below in Fig. 5.

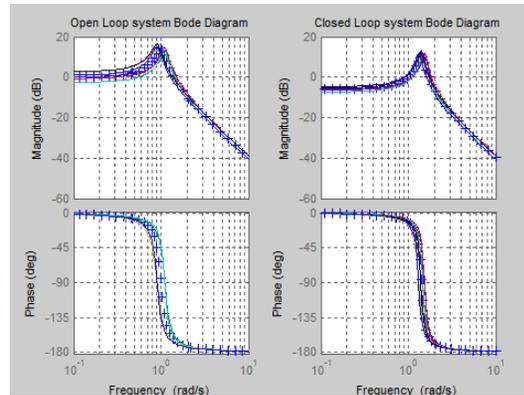


Fig. 5. Bode diagram (open loop Vs. closed loop)

The Nyquist diagram (open loop Vs. closed loop) is shown below in Fig. 6.

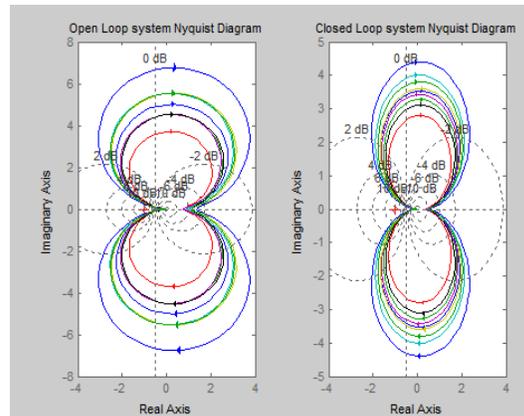


Fig. 6. Nyquist diagram (open loop Vs. closed loop).

In Fig. 7, the convex hull is presented and hence used to find the roots bounds on interval matrix as shown in Fig. 8, and using convex hull is reducing the level of computations that is involved in such problems as many points can be ignored as long as it is located inside the convex hull. Also it can be noticed that the system is stable since the symmetric bounds of eigenvalues are located on the left half of x- axis.

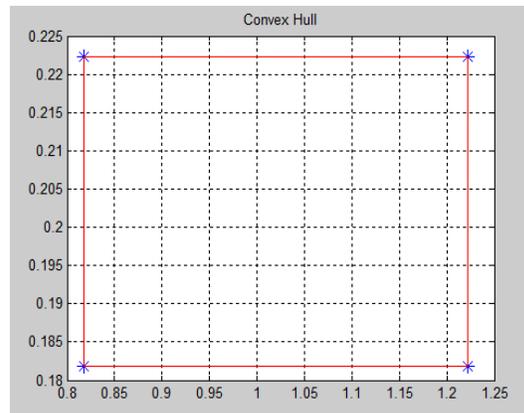


Fig. 7. Convex Hull

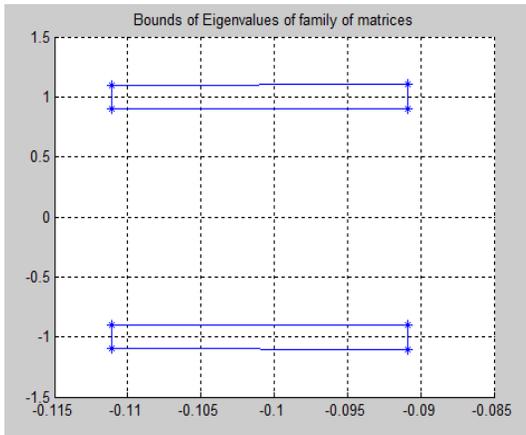


Fig. 8. Roots bounds of interval matrix

IV. EXAMPLE 2: RLC CIRCUIT

RLC circuit is an electrical circuit consisting of a resistor, an inductor, and a capacitor, connected in series or in parallel. The RLC part of the name is due to those letters being the usual electrical symbols for resistance, inductance and capacitance respectively. The circuit forms a harmonic oscillator for current and will resonate in a similar way as an LC circuit will. The main difference that the presence of the resistor makes is that any oscillation induced in the circuit will die away over time if it is not kept going by a source. This effect of the resistor is called damping. The presence of the resistance also reduces the peak resonant frequency somewhat.

The three circuit elements can be combined in a number of different topologies and our case is as shown in Fig. 9

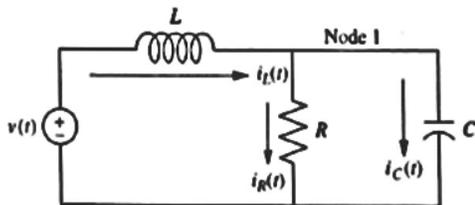


Fig. 9. RLC Circuit

By applying Kirchhoff's current and voltage derive the system differential equations as i_L and v_C are system state variables

$$\frac{dv_C}{dt} = -\frac{1}{RC}v_C + \frac{1}{C}i_L$$

$$\frac{di_L}{dt} = -\frac{1}{L}V_C + \frac{1}{L}V(t)$$

From the above equations the state space representation of this circuit is obtained as follows

$$\begin{bmatrix} \dot{v}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -1/(RC) & 1/C \\ -1/L & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} v(t)$$

$$i_R = [1/R \quad 0] \begin{bmatrix} v_C \\ i_L \end{bmatrix}$$

Also the systems parameters are presented below in table II

TABLE II. THE PHYSICAL PARAMETERS FOR RLC CIRCUIT

R	Resistance	1.000 Ω
L	Inductance	0.002 H
C	Capacitance	0.005 F

With 10% variation in Inductance and Capacitance with constant Resistance, the interval parameters are as follows:

$$L=[0.001 \quad 0.003]$$

$$R=1.0$$

$$C=[0.004 \quad 0.006]$$

Using the Mat-lab the $2^3 = 8$ sum matrices were generated from the uncertain system A matrix new matrices as follows

$m1 =$ $\begin{bmatrix} -250.0 & 166.7 \\ -1000.0 & 0 \end{bmatrix}$	$m2 =$ $\begin{bmatrix} -250.0000 & 166.6667 \\ -333.3333 & 0 \end{bmatrix}$
$m3 =$ $\begin{bmatrix} -250 & 250 \\ -1000 & 0 \end{bmatrix}$	$m4 =$ $\begin{bmatrix} -250.0000 & \\ 250.0000 & \\ -333.3333 & 0 \end{bmatrix}$
$m5 =$ $\begin{bmatrix} -166.7 & 166.7 \\ -1000.0 & 0 \end{bmatrix}$	$m6 =$ $\begin{bmatrix} -166.6667 & 166.6667 \\ -333.3333 & 0 \end{bmatrix}$
$m7 =$ $\begin{bmatrix} -166.7 & 250.0 \\ -1000.0 & 0 \end{bmatrix}$	$m8 =$ $\begin{bmatrix} -166.6667 & 250.0000 \\ -333.3333 & 0 \end{bmatrix}$

And its corresponding eight (8) polynomials were computed (by matlab) are as follows:

$$po1 = 1.0e+05 * \begin{bmatrix} 0.0000 & 0.0025 & 1.6667 \end{bmatrix}$$

$$po2 = 1.0e+04 * \begin{bmatrix} 0.0001 & 0.0250 & 5.5556 \end{bmatrix}$$

$$po3 = 1.0e+05 * \begin{bmatrix} 0.0000 & 0.0025 & 2.5000 \end{bmatrix}$$

$$po4 = 1.0e+04 * \begin{bmatrix} 0.0001 & 0.0250 & 8.3333 \end{bmatrix}$$

$$po5 = 1.0e+05 * \begin{bmatrix} 0.0000 & 0.0017 & 1.6667 \end{bmatrix}$$

$$po6 = 1.0e+04 * \begin{bmatrix} 0.0001 & 0.0167 & 5.5556 \end{bmatrix}$$

$$po7 = 1.0e+05 * \begin{bmatrix} 0.0000 & 0.0017 & 2.5000 \end{bmatrix}$$

$$po8 = 1.0e+04 * \begin{bmatrix} 0.0001 & 0.0167 & 8.3333 \end{bmatrix}$$

And step responses for open and closed loop are shown in figures 10 and 11 respectively.

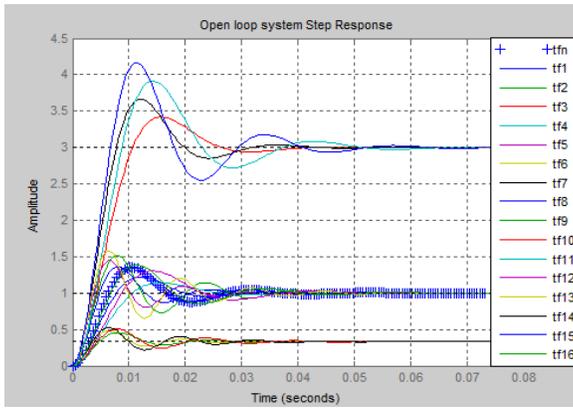


Fig. 10. Open Loop system step response

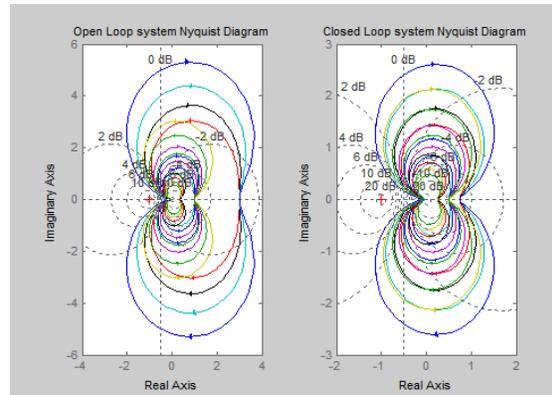


Fig. 13. Nyquist Diagram(open and Closed loop) is

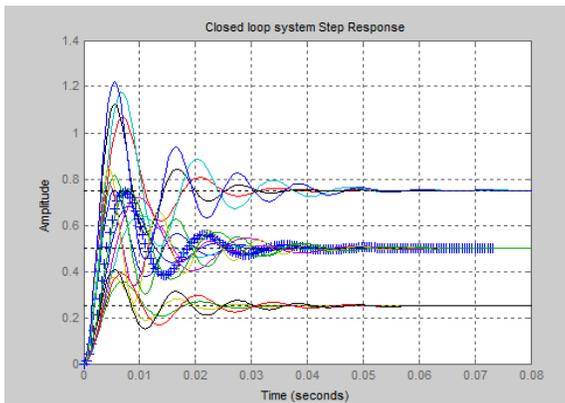


Fig. 11. Closed loop system step response

While the system Bode diagram (open loop Vs. closed loop) is shown in Fig. 12.

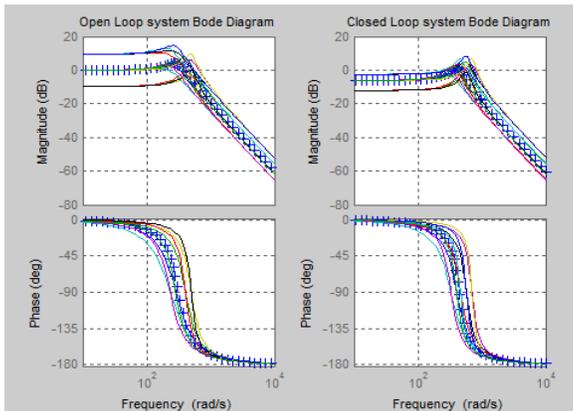


Fig. 12. Bode diagram (open loop Vs. closed loop)

Also the Nyquist Diagram (open and Closed loop) is illustrated in Fig. 13.

Finally the electrical circuit convex hull demonstrated in Fig. 14 which was used to locate the roots bounds as plotted in

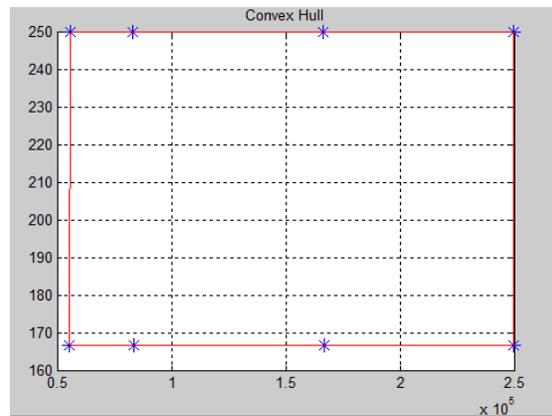


Fig. 14. Convex hull

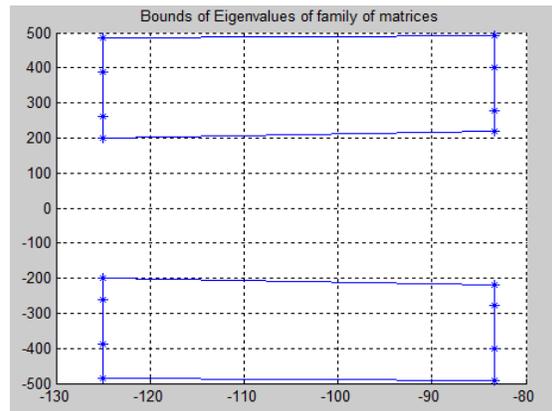


Fig. 15. Root bounds of interval matrix

As these symmetrical bounds clearly confirm the stability of the electrical interval circuit system.

V. CONCLUSION AND FUTURE WORK

In this paper the stability behavior of mechanical and electrical systems with uncertain parameters were modeled with robust control Matlab Toolbox®3. A good result was obtained as demonstrated in the uncertain mechanical and electrical examples. The computational time and efforts for determining the stability for interval problems (uncertain parameters) is very excessive, therefore as future work parallel algorithms and supercomputers are highly recommended in handling such problems, also this paper hoped to extended and be used as ground foundation to other applications such solar, thermal and wind as they suffer from disturbances and uncertain circumstances.

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AUTHORS PROFILE



Dr. Mohammed T. Hussein, Professor of Electrical Engineering joined the department of Electrical and Computer engineering at Islamic university of Gaza on August 2003. Dr. Hussein was named Director of e-Learning Center on November 1, 2003. Prior to this appointment he served as a department Head of Engineering Technology in College of Engineering at Prairie View A&M University, Texas. Dr. Hussein earned a Ph.D. degree in electrical engineering from Texas A&M University, College Station, Texas, USA. Dr. Hussein is a registered professional engineer (P.E.) in the State of Texas. Dr. Hussein worked for Motorola Inc., in Tempe, Az., and Oak

Ridge National Laboratory in state of Tennessee. His research interests include robust control systems, computer algorithms and applications, and e-Learning. Dr. Hussein holds scientific and professional memberships in IEEE(SM), Eta Kappa Nu, and Tau Beta Pi. He is the recipient of numerous national, state, university, college, and departmental awards including "Who's Who among America's Best Teachers" on 2000, "Marquis Who's who among World Leaders" on 2010, and "Teaching Award" in the College of Engineering. Dr. Hussein was nominated and selected on 2003 as an evaluator for Accreditation board for Engineering and Technology (ABET), USA. Dr. Hussein spent summer 2008 as a DAAD visiting Professor at Berlin Technical University, Germany, and on 2009 was selected as academy Fellow, Palestine Academy for Science and Technology.