A New Internal Model Control Method for MIMO Over-Actuated Systems

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***Abstract*—A new design method internal model control is proposed for multivariable over-actuated processes that are often encountered in complicated industrial processes.**

Due to the matrix that is adopted to describe over-actuated system is not square, many classical multivariable control methods can be hardly applied in such system. In this paper, based on method of virtual outputs, a new internal model control method is proposed.

The proposed method is applied to shell standard control problem (3 inputs and 2 outputs). The simulation results show that the robust controller can keep the set inputs without overshoot, steady state error, input tracking performance and disturbance rejection performance, the results are satisfactory have proved the effectiveness and reliability of the proposed method.

Keywords—internal model control (IMC); over-actuated multivariable system; inverse model; method of virtual outputs; disturbances rejections, stability; state error

I. INTRODUCTION

The multi-input and multi-outputs (MIMO) over-actuated systems exist in industrial and such are a difficult problem in the control [12], [21]. The multivariable over-actuated system is non-square system with the number of inputs is superior to that of outputs [7].

A simple of controlling the MIMO non-square system is transform it into a square system by adding or deleting variables. But adding variables will increase the control cost, while deleting variables reduce the quality of control by reason of missing information and may even make the process unstable between the credible introduction of right half plane [1], [8], [9], [16] .

The Internal Model Control (IMC) which due to its simplicity, excellent robustness, and better control performance shows the capacity to solve the control problems of the multivariable systems. If we combine IMC with multivariable control system, it could be a capable way to solve the control difficulty of multivariable systems [11].

On the basis of the IMC principle, the design of a controller for a square system is generally based on the inverse matrix of system [3], [4], [20]. Nevertheless, considering non-square processes, IMC cannot be applied exactly, as it cannot obtain the traditional sense of inversion [5], [6].

In recent years, many researchers adopted finding the robust controller by internal model control.

Seshagiri [2] designed a PI controller as a Smith delay compensator for non-square system with multiple time delays. This method achieves static decoupling because it is based on the pseudo inverse of the steady-state gain matrix of non-square systems. Only using the steady-state information of the systems will lead to the limitation of control performance.

Chen [17] a modified the IMC el for non-square systems by inserting compensated to remove the terms unrealizable factoring there for obtained from the controller. The objective the controller parameter is to achieve tracking performance and robustness.

Chen [9] proposed a new method using Internal Model Control and smith controller between design a PI controller for multivariable non-square systems with transfer function elements consisting of first order and time delay. The problem of this method is no analysis of load disturbance performance, and the decoupling effect is poor than dynamic decoupling.

Quan [12] proposed a new NERGA based on internal model control method for non-square system. This method calculate the inverse of the matrix, the model controller is designed based on the model of squared subsystem. But when building the subsystem controller, that is to say we eliminate variables the global system will reduce the quality and control performance.

Liu [10] proposed a method a modified two-degrees-of-freedom internal model control method for non-square systems with multiple time delays and right-half-plane zeros. This method, pseudo-inverse is introduced to describe the internal model controller, and an appropriated closed-loop transfer function is designed to eliminate the impracticable factors of the derived controller.

Jin [18] proposed a design method of decoupling IMC for non-square processes with multiple time delays. The method can achieve a realizable decoupling controller of non-square processes by inserting some compensated terms. At the same time, based on the relative normalized gain array, an equivalent transfer function matrix is acquaint to approximate the pseudo-inverse of the process transfer function matrix.

This paper presents a new technique to enquire into the IMC control design for multivariable over-actuated system. In the controller design procedure a simple method designed to uses virtual outputs method [1]. Finally, this method is applied in a system with 3 inputs than two outputs; the simulation results show that the proposed method had good performance of tracking ability and strong performance.

This paper is organized as follows. Section II presents a generality on the internal model control strategy of multivariable system. Section III proposes the design method of the controller for the internal model control for multivariable over-actuated system. In section IV, an example is employed to illustrate the effectiveness of the proposed controller. Some conclusions are drawn in section V.

II. STRUCTURE OF IMC FOR MIMO PROCESSES

The internal model control (IMC) found wide acceptance in process control system, due to be simplicity, excellent robustness, and good control performance, shows the strong vitality to solve the control problems of multivariable non-square and square systems.

The internal model control structure of multivariable process as shown in Fig. 1. Where G(s), M(s), C(s) and $G_v(s)$ represent the transfer functions of the process, the process model, IMC controller and disturbance respectively; y and y_m are the outputs vectors of the process and its model, respectively; r is the input vectors of the process; u represents the control input signal; v is the disturbance.

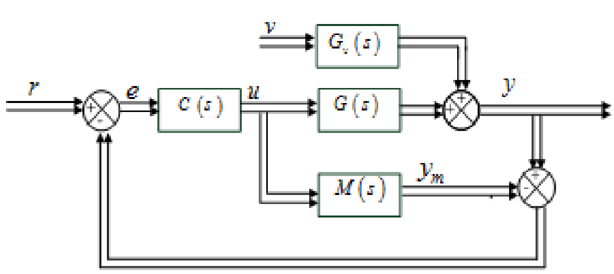

Fig. 1. Structure of Internal model control (IMC)

Where, all elements in G(s), $G_m(s)$ are stable [19].

The equation of input, output and disturbance is given by [19]:

$$y(s)=G(s)\left[I+C(s)\left(G(s)-M(s)\right)\right]^{-1}C(s)r(s) + \left[I-G(s)\left[I+C(s)\left(G(s)-M(s)\right)\right]^{-1}C(s)\right]G_v(s)v(s) \quad (1)$$

When the model is perfect, the model process is given by:

$$M(s)=G(s) \quad (2)$$

The expression of output transfer function can be obtained to equation (3):

$$y(s)=G(s)C(s)r(s)+\left[I-G(s)C(s)\right]G_v(s)v(s) \quad (3)$$

The MIMO transfer functions for the process G(s) with 'n' inputs and 'm' outputs (m<n) is considered as [7]:

$$G(s)=\begin{pmatrix} G_{11}(s) & G_{12}(s) & \cdots & G_{1n} \\ G_{21}(s) & G_{22}(s) & \cdots & G_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1}(s) & G_{m2}(s) & \cdots & G_{mn}(s) \end{pmatrix} \quad (4)$$

Transfer function of the controller is :

$$C(s)=\begin{pmatrix} k_{11}(s) & k_{12}(s) & \cdots & k_{1m} \\ k_{21}(s) & k_{22}(s) & \cdots & k_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ k_{n1}(s) & k_{n2}(s) & \cdots & k_{nm}(s) \end{pmatrix} \quad (5)$$

We consider the IMC configuration that is stable for process, the model of the process and the IMC controller.

In IMC, the synthesis of a controller which is equal to the inversion of the model system is paramount to ensure perfect follow instructions. But, the manner of direct inversion is practically impossible for over-actuated systems. To remedy this problem, it is proposed to develop a method of inversion in the case of over-actuated multivariable linear systems.

III. CONTROLEUR DESIGN

A. Structure of the controller to a over-actuated multivariable system

For MIMO non-square system, the input number is unequal to the output number. There are two types of non-square systems: the under-actuated system where the number of inputs is inferior than the number of outputs (m>n) and the system over-actuated system the number of inputs is superior to that of outputs (m<n) [7].

In this paper we will take an interest in the over-actuated system, and we will follow a methodology to design our controller for this case of system.

Using the method of virtual outputs of adding lines to the transfer matrix of the non-square system, up to have a square transfer matrix that can be reverse [1].

Regarding the virtual outputs that will add to the transfer matrix system, we will copy the outputs of the original system and the programmation part we will remove them. these virtual outputs will be used to make the square transfer matrix.

The transfer function matrix of the over-actuated system is then a rectangular matrix recess non-singular. If we extend the matrix G(s) to make it square. This amounts to consider ((n-m), n) further referred to as the word of virtual outputs.

Our system is represented by the following equation (11),

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \dots & G_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1} & G_{m2} & \dots & G_{mn} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix} \qquad (11)$$

The transfer matrix that will add to make the over-actuated system square is its size ((n-m), n). This matrix has the following form:

$$\begin{pmatrix} y_{m+1} \\ y_{m+2} \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} G_{m+1,1} & G_{m+1,2} & \cdots & G_{m+1,n} \\ G_{m+2,1} & G_{m+2,2} & \dots & G_{m+2,n} \\ \vdots & \vdots & \ddots & \vdots \\ G_{n1} & G_{n2} & \dots & G_{nn} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix} \qquad (12)$$

In the system simulation phase, we need to add a function block which eliminates (m-n) output added to visualize the non-square system output.

The simulation block diagram is given in Fig.3.

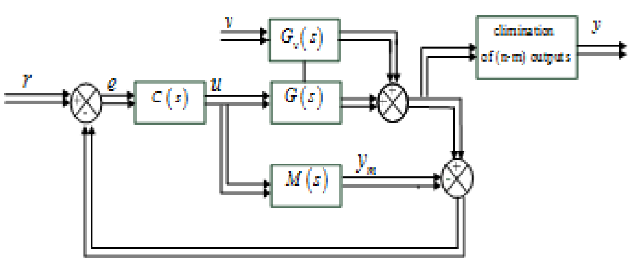

Fig. 2. IMC structure for multivariable over-actuated system

B. *Study of the stability of the proposed controller*

The IMC controller is present on Fig.2 by using the inversion method proposed [13], [14]. K_1 reversal of the matrix is an invertible square matrix; it must ensure the stability conditions of the controller.

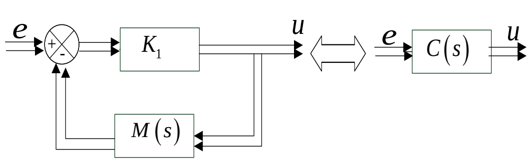

Fig. 3. Structure for model inversion

The expression of internal model controller can be obtained:

$$C(s) = \frac{K_1}{I_m + K_1 M(s)} \qquad (6)$$

the K_1 inversion matrix of the form $K_1 = a\,I_m, a \in \mathbb{R}^+$, I_m is the identify matrix is chosen sufficiently High then $\frac{1}{a}$ is sufficiently low there by approximating given by the equation [15]:

$$\frac{1}{\frac{1}{K_1} + M(s)} = M(s)^{-1} \qquad (7)$$

The stability of the structure proposed for the internal model depends of stability of the process control, of the model and of the controller C(s) respectively.

The controller C(s) can be written as follows [15]:

$$C(s) = \frac{t_{com}\left(I_m + K_1 M(s)\right) K_1}{\det\left(I_m + K_1 M(s)\right)} \qquad (8)$$

are *N (s)* and *D (s)* respectively represent the numerator and denominator of $\det\left(I_m + K_1 M(s)\right)$.

To ensure the stability of the regulator *C(s)*, it must be ensured that *N(s)* is a polynomial of Hurwitz. This means that the roots of N(s) must be strictly negative real parts. These roots can be located either using geometric methods such as root locus or algebraic methods such as of Routh criterion.

Given a model *M(s)* Stable, adequate choice of K_1 inversion matrix then ensures the stability of the regulator *C(s)*.

C. *Precision study of the system*

To ensure the accuracy of the system, that is to say a zero static error, check that [15]:

$$C(0) = \frac{1}{M(0)} \qquad (9)$$

With *C(0)* is the matrix of the static gains of the controller. It can be expressed as a function of the matrix of the static gains of the system *M (0)*. It is defined by the equation (10):

$$C(0) = \frac{K_1}{\left(I_m + K_1 M(0)\right)} \qquad (10)$$

It can be said the performance of the proposed controller, to ensure a perfect tracking of the reference input independently of external disturbances. This property can only be validated if we choose a sufficiently high.

IV. SIMULATION RESULTS

Consider a 2×3 stable over-actuated multivariable system; the transfer function of the model system is given as:

$$G(s)=\begin{pmatrix} \frac{s+2}{s^2+3s+4} & \frac{1}{s+3} & \frac{1}{s^2+2s+3} \\ \frac{s+1}{3s^2+3s+2} & \frac{3}{s+4} & \frac{1}{s^2+2s+1} \end{pmatrix} \qquad (13)$$

Where there are two controlled variables (y_1, y_2) and three manipulated variables (u_1, u_2, u_3).

The model is represented by the following transfer matrix function M(s), it is defined by the equation (14):

$$M(s)=\begin{pmatrix} \frac{s+2}{s^2+3s+4} & \frac{1}{s+3} & \frac{1}{s^2+2s+3} \\ \frac{s+1}{3s^2+3s+2} & \frac{3}{s+4} & \frac{1}{s^2+2s+1} \\ \frac{s+2}{s^2+3s+4} & \frac{1}{s+3} & \frac{1}{s^2+2s+3} \end{pmatrix} \qquad (14)$$

In our case, the chosen matrix K_1 is equal to $K_1 = 40 \times I_3$, to ensure the stability of the system G(s) for controlling.

Fig. 4 and Fig. 5 represents the evolution of the internal model controllers u_1(t) and u_2(t). The unit step responses of outputs y_1(t) and y_2(t) are shown in Fig.6 and Fig.7. From the analysis of Fig.6 and Fig. 7, it follows that the system control effect is satisfactory without overshoot, static error and the system has good traceability. The resulting outputs responses of the system are in order affirm the effectiveness of the proposed internal model controller.

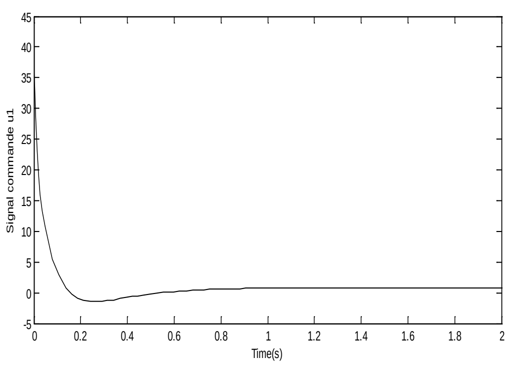

Fig. 4. The control input u

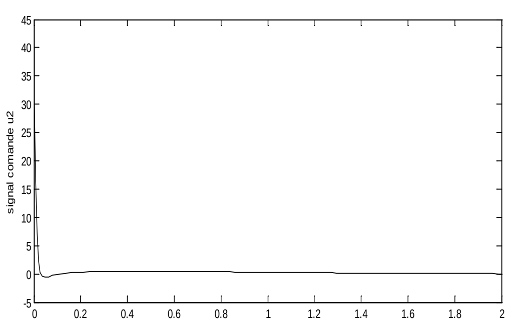

Fig. 5. The control input u_2

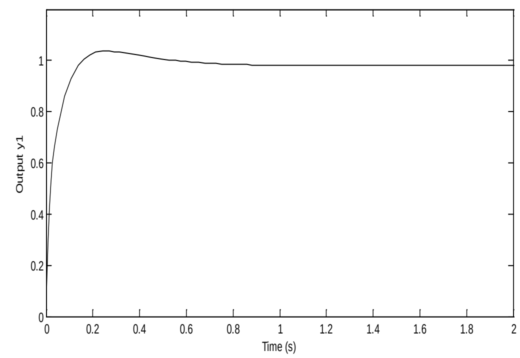

Fig. 6. The step response of output y1

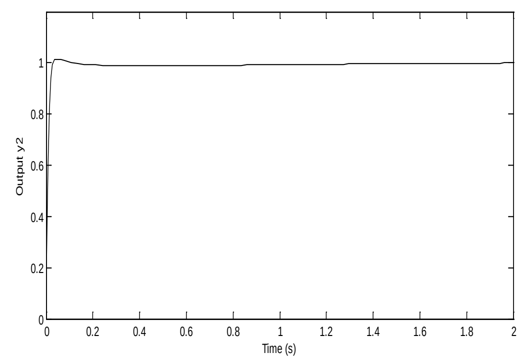

Fig. 7. The step response of output y2

The disturbance signal is expressed by equation (15)

$$G_v(s)=\begin{bmatrix} \frac{e^{-0.5s}}{s} & \frac{e^{-0.5s}}{s} & \frac{e^{-0.5s}}{s} \end{bmatrix}^T \qquad (15)$$

In order to assert the disturbance rejection capability of the system, assumed a step disturbance signal with magnitude 1 was added to the input 1, input 2 and input 3 at t=1s.

Fig. 8 and Fig. 9 represents the evolution of the internal model controllers u_1(t) and u_2(t).

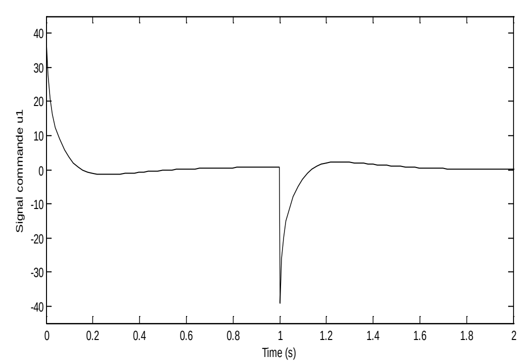

Fig. 8. The control input u_1 with disturbance

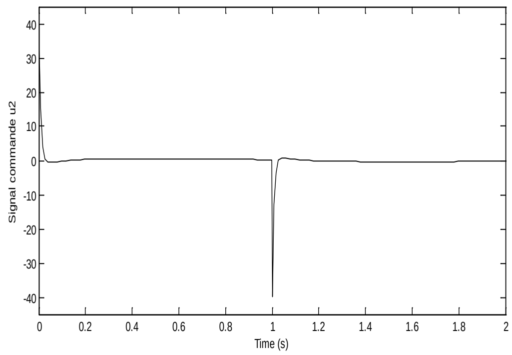

Fig. 9. The control input u_2 with disturbance

The simulation results of model perturbation responses of outputs $y_1(t)$ and $y_2(t)$ are given in Fig.10 and Fig.11. The simulation results show that the proposed method has disturbance rejection performance. It clearly shows that the set-point tracking and disturbance rejection are achieved and it offers robustness.

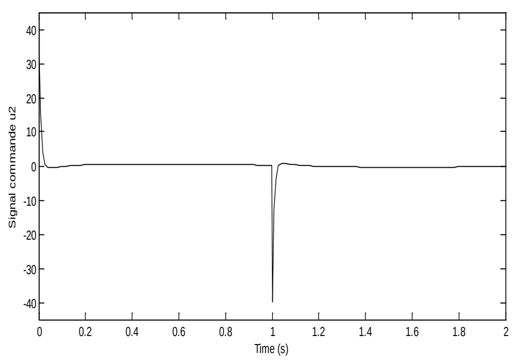

Fig. 10. The step response of output y1 with disturbance

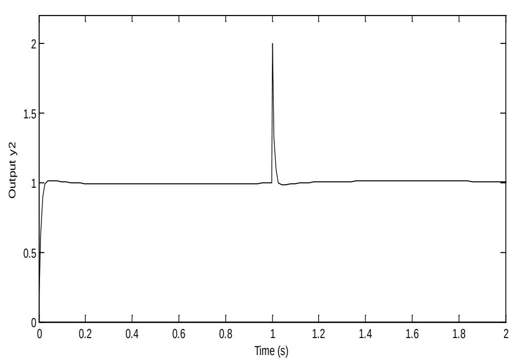

Fig. 11. The step response of output y2 with disturbance

V. CONCLUSION

For the multivariable system with the number of inputs is superior to that of outputs are habitually met in system industries, we proposed a new method of virtual outputs based on internal model control for over-actuated system has been presented in this paper. This novel method avoids the complex calculation; such as calculate the inverse of this matrix, the controller structure is simple.

The multivariable over-actuated system is not square, so the reverse is not possible with our method of virtual outputs is added, our system becomes square at that moment, and we can build our controller which is based on the reversal of the processes.

The simulation results show that this method proposed has the advantages of small overshoot, the set-point tracking controller and disturbance rejection performance. Meanwhile, better control performance and good robustness than other control methods of over-actuated systems.

Generally, this new method is simple, has robust performance and easy to implement in engineering processes.

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