WQbZS: Wavelet Quantization by Z-Scores for JPEG2000

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***Abstract*—In this document we present a methodology to quantize wavelet coefficients for any wavelet-base entropy coder, we apply it in the particular case of JPEG2000. Any compression system have three main steps: Transformation in terms of frequency, Quantization and Entropy Coding. The only responsible for reducing or maintaining precision is the second element, Quantization, since it is the element of lossy compression that reduces the precision of dequantized pixels in order to make quantized pixels more compressible. We modify the well-known dead zone scalar Quantization introducing Z-Scores in the process. Thus, Z-scores are expressed in terms of standard deviations from their means. Resultantly, these z-scores have a distribution with a mean of 0 and a standard deviation of 1, in this way we increase redundancies into the image, which produces a lower compression ratio.**

Keywords*—Z-Scores; Statistical Normalization; Wavelet Transformation; Scalar Quantization; Deadzone Quantization; JPEG2000

I. INTRODUCTION

One of the most important features of human beings is *Vision*, because is one of the most difficult sense to model, since not only involve mathematical models but also experience passages of the live of the a person, which can be different of each one. So, when a light ray enters into our eyes launches a highly complex process, which finalizes in the brain specifically into the visual cortex. Thus, scientists in this field intent to give a mathematical response of some of these features of the Human Visual System(HVS).

Digital image compression is a research topic for many years until today and a number of image compression algorithms is created for different applications. The JPEG2000 is a standard that tries to reduce the rate of stored pixels regarding its distortion rate, taking in account objective and subjective image quality. Several works have been demonstrated that the overall performance of JPEG2000 is superior to existing standards, as well as to supply functionality [1]. Figure 1 shows the comparison of bit rate of (a) JPEG and (b) JPEG2000 image coders, tested with the 24-bit and 512×512 pixel Color Image *Lena*. The results for this particular case and bit rate show that JPEG2000 is better in 2.63 dB, which is an important improvement.

However, JPEG2000 barely provide relevant features of the human visual system, since for removing pixels, in order to find more redundancies inside the image, JPEG2000 mainly applies criteria of the Information Theory such as thresholds, for instance. This this lack of information introduces artifacts into the recovered image, which are notorious at high compression rates, that is because many the most visible pixels regarding its perceptual significance have been eliminated.

In addition, JPEG2000 s an image compression standard approach, which was proposed by ISO/IEC. from previous standards, also, it was created as a framework where the image compression system can have the behavior of an image processing algorithm. The decision on several important compression features such as quality or resolution are created after the generation of the coded codestream. Thus, JPEG2000 decoded many image algorithms from a single coded file, give as a result different chances for coded domain processing. As in any compression or coding system, Quantization procedure is one of the critical steps of JPEG2000 image coding and decoding algorithm. Many of the desirable properties of the JPEG2000 standard contain manly two quantization methods, such as Embedded Scalar Quantization.

In this paper, we provide an overview of well-known methods in addition to propose a new one WQbZS. This paper is organized as follows. In Section 2, an introduction to quantization methods used in JPEG2000 is presented. Section 3 describes the WQbZS algorithm implemented in JPEG2000, in addition, we define the statistical relevancy of the Z-Scores, and Section 4 provides the experimental results.

II. A BRIEF DESCRIPTION OF JPEG2000 QUANTIZATION AND ITS ENVIRONMENT

A. Image Compression System

General Theory of Systems defines $information$ as $-entropy$, i.e. $negentropy$. Let us define first the concept of *entropy*, which is the tendency a system has when it tend to disintegrate by itself or by external factors[2]. Thus, entropy means the grade of disorder of a system. In the same way, a recovered image should have almost the same total entropy as the original one, but using less bits per every pixel. That is, a compressed image should have more entropy per bit than the original one. In addition, let us to mention that the main objective of recent image compression systems is to increase redundancies of images, understanding that some frequencies are redundant. These redundancies inside frequencies can be obtained by statistical procedures or the estimation of visual irrelevancies[3, Sec. 1.2].

A system is generally defined as a subset mainly composed by three subsystems: an input, a process, and an output, in

(a) JPEG: 0.22bpp, PSNR = 27.39 dB

(b) JPEG2000: 0.22bpp, PSNR = 30.03 dB

Figure 1: Comparison of bit rate of JPEG2000 and JPEG image coders, tested with the 24-bit Color Image *Lena*.

some cases we can include a fourth subsystem feedback, these subsystems define a cybernetic model, which is depicted in Figure 2. Hence, any system is defined as a set of the three o four elements standing in interrelation among themselves and also with the environment of the system.

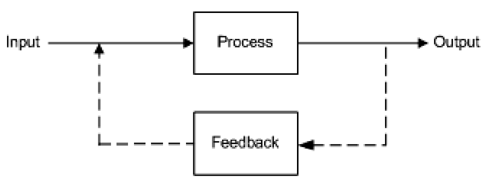


Figure 2: Description of *System* according to the General Theory of Systems.

Subsystems *Process* and *Feedback* have no relation, but *Feedback* is employed in order to adjust some characteristics or to evaluate how efficient is the *Process*. In the same way, an image compression coder is described as a general system as follows, Figure 3:

- *Input*: Original image considered with infinite and unquestionable quality $f(i, j)$;
- *Process*: Set of subsystems, these are: Forward Transformation, Quantization, Entropy Coding, Entropy Decoding, Inverse Quantization and Inverse Transformation;
- *Output*: Recovered image $\hat{f}(i, j)$;
- *Feedback*: Assessment of the possible distortion between original and recovered images, in order to measure the efficiency of the image compression system.

B. *Dead-zone Uniform Scalar Quantizer in JPEG2000*

Marcellin et.al. give us a general overview in [4] of the uniform scalar quantizer. This kind of quantization process is described as a mathematical model that maps every pixel or coefficient into a particular energy, which maintain the entropy but reduces the compression ratio. This way, this quantization values are uniformly distributed by rage known as a *QStep* with size Δ, this is fulfilled across the range, except when the energy of the pixel is quantized to zero, which is known as Dead-Zone. The width of this Zone extends from $-\Delta$ to $+\Delta$. Thus, a dead-zone can be defined as the quantization range around 0, which is twice the size of Δ, namely all the coefficients or pixels lower than $|\Delta|$ cannot be recovered in the dequantization process.

Thus, in a given wavelet plane ω_s^o , with spatial frequency s and spacial orientation o, and a particular *QStep* size Δ_s^o is used to quantize all the coefficients inside a wavelet decomposition. Hence a particular *QStep* is defined as follows:

$$\overline{c_{i,j}} = sign(c_{i,j}) \left\lfloor \frac{|c_{i,j}|}{\Delta_s^o} \right\rfloor \tag{1}$$

where $c_{i,j}$ is the original wavelet coefficient value, $sign(c_{i,j})$ denotes the sign of $c_{i,j}$ and $\overline{c_{i,j}}$ is the resulting *QStep* coefficient. Figure 4 illustrates such a *QStep* size Δ, here vertical lines indicate the thresholds of the quantization ranges and heavy points show the recovered coefficient.

The inverse quantizer or the recovered $\widehat{c_{i,j}}$ is given by

$$\widehat{c_{i,j}} = \begin{cases} (c_{i,j} + \delta)\Delta_s^o, & c_{i,j} > 0 \\ (c_{i,j} - \delta)\Delta_s^o & c_{i,j} < 0 \\ 0, & c_{i,j} = 0 \end{cases} \tag{2}$$

where δ is a parameter that intents to reconstruct $c_{i,j}$ at the center of a given quantization interval and varies form 0 to 1.

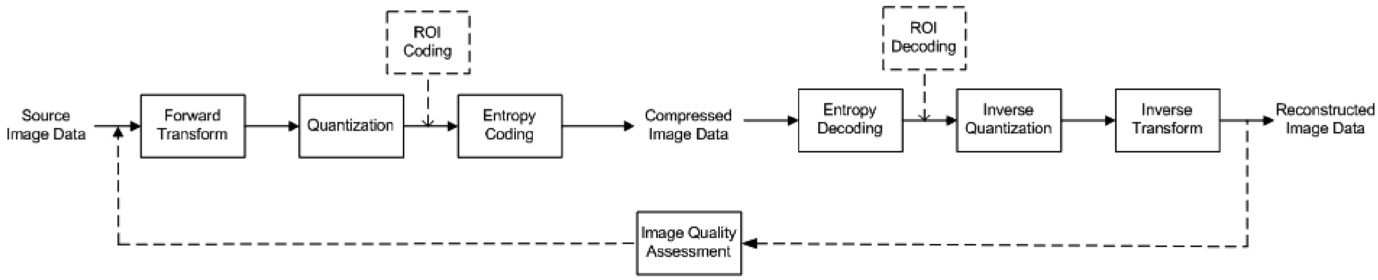


Figure 3: Scheme in order to define any Image Compression System.

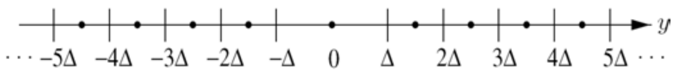


Figure 4: Dead-zone uniform scalar quantizer with *QStep* size Δ.

The International Organization for Standardization recommends to adopt the middle point in order to reconstruct $c_{i,j}$, setting $\delta = 0.5$ [1]. Pearlman and Said in [5] find that $\delta = 0.375$ obtains better results, especially for high frequency wavelet planes. when $-\Delta < y < \Delta$, the quantizer level and reconstruction value are both 0. Since it is known that many coefficients in a wavelet transform are close to zero (usually those of higher frequencies), it means that they can be on the dead-zone, namely $\widehat{c_{i,j}} = 0$.

It is important to realize that Quantization Process is the only subsystem that induces degradations into the image compression system, so when a wavelet plane ω_s^o is quantized is because $f(i,j)$ would be losslessly compressed, but the induction of $|\Delta| \neq 1$ causes a lossy compression, on the contrary when $|\Delta| = 1$ it causes lossless compression.

III. WQbZS Algorithm

A. Methodology

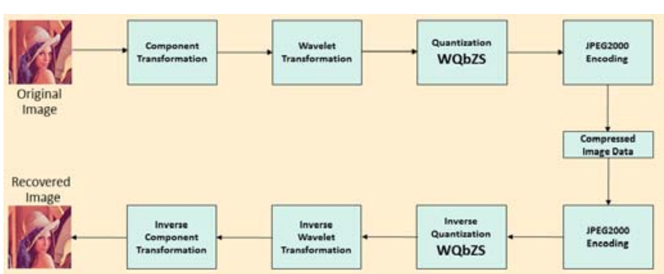


Figure 5: WQbZS Algorithm.

Figure 5 shows the present proposal, which is divided in 2 main stages with 3 steps each one:

1) Coding a JPEG2000 Image.
   - Component Transformation,
   - Wavelet Transform, and
   - Definition of Z-Scores for JPEG2000 Quantization.
2) Decoding a JPEG2000 Image.
   - Inverse Component Transformation,
   - Inverse Wavelet Transform, and
   - Definition of Z-Scores for JPEG2000 Inverse Quantization.

B. Component Transformation

For widespread applications , Data Compression systems of Natural Images, like JPEG2000, usually code color images. These images are numerically represented in several Chromatic Spaces both receptional representations, such as RGB or $CMYK$, and post-receptional representations, such as YC_bC_r, YCM, or HSB, being RGB the most commonly used along with YC_bC_r.

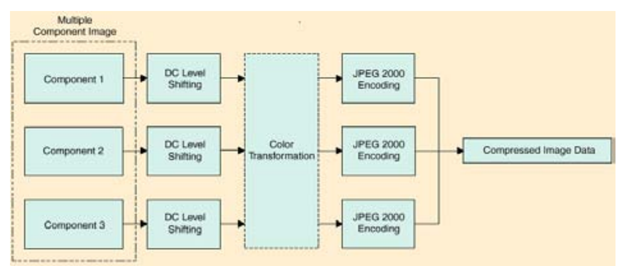


Figure 6: JPEG2000 Multiple Component Encoder.

In this way, a color image represented by a RGB color space, is decomposed into the same number of wavelength that in our cones in our retina can perceive, namely Red, Green, and Blue color components or cones. Figure 6 shows a special implementation of the post-receptional color space YC_bC_r, when JPEG2000 performs a chromatic coding, a complete encoding is performed at each color component. R, G and B color channels are numerically more dependent than Y, C_r and C_b, thus the chrominance channels are independently coded at lower size than luminance one in order to get better compression rates [6].

The JPEG2000 standard considers both Reversible Component Transformation (RCT) and Irreversible Component Transformation(ICT) [1, Annex G]. For lossy coding or with some degradations is employed an ICT, which makes use of the the 9/7 wavelet transform also irreversible. Thus, the forward and inverse filters to compute a 9/7 wavelet transform are estimated by the Equation 3 and 4, respectively [7], [3].

$$\begin{bmatrix} Y \\ C_b \\ C_r \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.16875 & -0.33126 & 0.5 \\ 0.5 & -0.41869 & -0.08131 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1.0 & 0 & 0.114 \\ 1.0 & -0.34413 & -0.71414 \\ 1.0 & 1.772 & 0 \end{bmatrix} \begin{bmatrix} Y \\ C_b \\ C_r \end{bmatrix}. \quad (4)$$

RCT is commonly employed not only for lossy compression but also for lossless encoding, along with the 5/3 wavelet transform, which is also reversible. The forward RCT transformation is computed by Equation 5 while the inverse by the Equation 6.

$$\begin{bmatrix} Y \\ C_b \\ C_r \end{bmatrix} = \begin{bmatrix} \left\lfloor \frac{R+2G+B}{4} \right\rfloor \\ R-G \\ B-G \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} Y - \left\lfloor \frac{C_r+C_b}{4} \right\rfloor \\ C_b+G \\ C_r+G \end{bmatrix} \begin{bmatrix} Y \\ C_b \\ C_r \end{bmatrix} \quad (6)$$

IV. Wavelet Transform

The original image $\mathcal{I}$ used by JPEG2000 is separated into different spatial frequencies and orientations using a multi level or multiresolution Direct Wavelet Transform (DWT) either Reversible or Irreversible [8], [9], by each channel. Thus $\mathcal{I}$ is separated in different set of planes or wavelet planes ω or spatial frequencies, where each wavelet plane has details at a given spatial resolutions and it is defined as follows:

$$DWT\{\mathcal{I}(w)\} = \sum_{s=1}^{n} \sum_{o=v,h,d} \omega_s^o + c_n(t) \quad (7)$$

where $s = 1, \ldots, n$, n the number of wavelet planes (in frequency domain) and $c_n(t)$ the residual plane, it is important to mention that this plane is the only part of the DWT which is the time or pixel domain. Spatial orientation is represented by $o = v, h, d$ i.e. vertical, horizontal or diagonal details, respectively.

A DWT filters each row and column of $\mathcal{I}(w)$ with a high-pass and low-pass filters, respectively. This algorithm yields in the duplication of samples, so the resultant image is downsampled by 2 both for column and rows, thus the number of sample remains in the same amount.

The Reversible Direct Wavelet Transform is performed by a 5/3 filter. Analysis and its respective synthesis filters are exposed in Table I. While, 9/7 filter is employ to perform a Irreversible Direct Wavelet Transform and its analysis and synthesis filters are depicted by Table II.

It is invariant if the columns or the rows of the Y, C_r or C_b channels are processed first.

The number of filtering levels or stages n of wavelet planes, depends directly on its usage. Notwithstanding that, some authors report that the best results are gotten with $n = 3$ [10], if it is taking into account the trade-off between image quality and compression ratio,.

Figure 7 depicts the Irreversible Direct Wavelet Transform of the Y component applied in the image *Peppers* with $n = 3$.

Table I: 5/3 Analysis and Synthesis Filter.

Analysis Filter		
i	Low-Pass Filter $h_L(i)$	High-Pass Filter $h_H(i)$
0	6/8	1
± 1	2/8	-1/2
± 2	-1/8	
Synthesis Filter		
i	Low-Pass Filter $h_L(i)$	High-Pass Filter $h_H(i)$
0	1	6/8
± 1	1/2	-2/8
± 2		-1/8

Table II: 9/7 Analysis and Synthesis Filter.

Analysis Filter		
i	Low-Pass Filter $h_L(i)$	High-Pass Filter $h_H(i)$
0	0.6029490182363579	1.115087052456994
± 1	0.2668641184428723	-0.5912717631142470
± 2	-0.07822326652898785	-0.05754352622849957
± 3	-0.01686411844287495	0.09127176311424948
± 4	0.02674875741080976	
Synthesis Filter		
i	Low-Pass Filter $h_L(i)$	High-Pass Filter $h_H(i)$
0	1.115087052456994	0.6029490182363579
± 1	0.5912717631142470	-0.2668641184428723
± 2	-0.05754352622849957	-0.07822326652898785
± 3	-0.09127176311424948	0.01686411844287495
± 4		0.02674875741080976

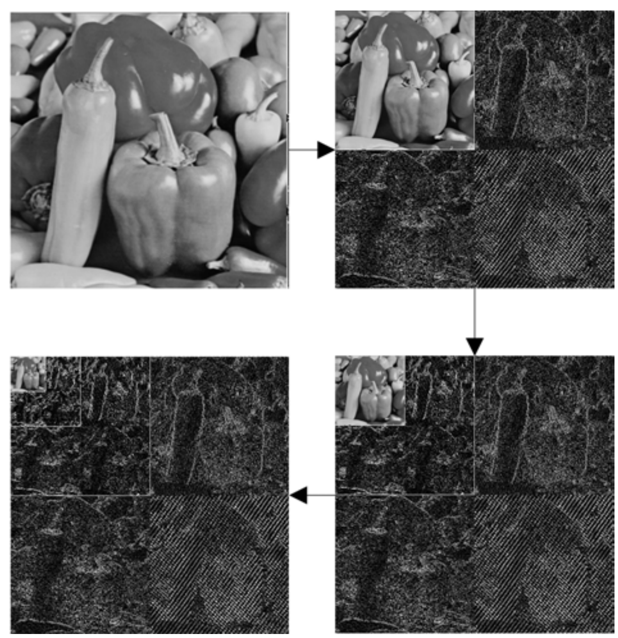


Figure 7: Three stages to decompose an image by means of a Direct Wavelet Transformation for *Peppers* image.

A. Definition of Z-Scores for JPEG2000 Quantization

A Z-score is a numerical measurement of a relationship of samples to the mean in a group of values. If a Z-score is 0, it represents the score is identical to the mean score. Exist another implementations of Z-scores and in some literature they are commonly known as the Altman Z-score. Edward Altman, a professor at New York University, developed and introduced the Z-score formula in the late 1960s as a solution to the time-consuming and somewhat confusing process investors had to undergo to determine how close to bankruptcy a company was[11], for instance.

In this work we consider the samples as Intensity values either Y, C_r or C_b channel in wavelet domain. So, in this case Z-scores can be positives or negatives, with a positive value indicating the score is above the mean of the coefficients of wavelet planes and a negative score indicating it is below its mean. Positive and negative scores also reveal the number of standard deviations the score is either above or below the mean, namely if it is easy to increase the redundancies around the average of frequencies in different spacial orientations.

Z-scores also reveal if a wavelet decomposition is typical for a specified Image $\mathcal{I}(w)$ or if it is atypical. In addition to this, Z-scores also make it possible for analysts to adapt scores from various Images $\mathcal{I}(w)$ to make scores that are compared to one another accurately.

In this way, a z-score or standard score indicates how many standard deviations a coefficient ω_s^o is from the mean. The general Equation for estimate a z-score is calculated from Equation 8.

$$Z_s^o = \frac{\omega_s^o - \mu_s^o}{\sigma_s^o} \tag{8}$$

where Z is the Z-score, ω_s^o is the value of the coefficient in the wavelet domain, μ_s^o is the population mean, and σ_s^o is the standard deviation. s the number of a particular wavelet plane of ω^o in addition spatial orientation is represented by $o = v, h, d$ i.e. vertical, horizontal or diagonal details, respectively.

Z-scores of the the coefficients ω_s^o of a Direct Wavelet Transform can be interpreted as follows:

- A Z-score less than 0 represents a set of coefficients ω_s^o less than its mean μ_s^o.
- A Z-score greater than 0 represents a set of coefficients ω_s^o greater than its mean μ_s^o.
- A Z-score equal to 0 represents a set of coefficients ω_s^o to its μ_s^o.
- A Z-score equal to 1 represents a set of coefficients ω_s^o that is 1 standard deviation σ_s^o greater than its mean μ_s^o; a Z-score equal to 2, 2 standard deviations σ_s^o greater than its mean μ_s^o; etc.
- A Z-score equal to -1 represents a set of coefficients ω_s^o that is 1 standard deviation σ_s^o less than the mean μ_s^o; a z-score equal to -2, 2 standard deviations σ_s^o less than the meanμ_s^o; etc.

As the number of coefficient in the set ω_s^o is very large, about 68% of the set of coefficients ω_s^o have a Z-score between -1 and 1; about 95% have a Z-score between -2 and 2; and about 99% have a Z-score between -3 and 3.

In this way, when we introduce a Z-score expressed by Equation 8 to Equation 1, we propose Equation 9, which is a Z-score for JPEG2000 Quantization.

$$\overline{\omega_s^o} = sign(\omega_s^o) \left\lfloor \left| \frac{\omega_s^o - \mu_s^o}{\sigma_s^o} \right| \right\rfloor \tag{9}$$

Finally, we introduce $\overline{\omega_s^o}$ to a general decomposition expressed in Equation 7. Thus, we propose Equation 10 in order to quantized any wavelet coefficient subset ω_s^o, which will be encoded by JPEG2000.

$$\mathcal{I}(w) = \sum_{s=1}^{n} \sum_{o=v,h,d} sign(\omega_s^o) \left\lfloor \left| \frac{\omega_s^o - \mu_s^o}{\sigma_s^o} \right| \right\rfloor + c_n(t) \tag{10}$$

V. Experimental Results

We test our algorithm in two different ways. By one hand, we perform a test with a well-known image, *Lena* 8. By the other hand, we test our methodology with two important Image Databases, *CMU* and Image Databases*CSIQ*.

Nowadays, Mean Squared Error (MSE) is still the most used objective performance metrics and several quality assessments are based on it, Peak Signal-to-Noise Ratio (PSNR) is the best example of it. But some authors like Wang and Bovik in [12], [6] consider that MSE is a poor device to be used in quality assessment systems. In this work we use PSNR to compere our results regarding the ones obtained by the standard.

In this way, $f(i,j)$ and $\hat{f}(i,j)$ represent two images, which we want to compare and the size of them is the same. Then, $f(i,j)$ is the original reference image considered with unquestionable and perfect quality, in addition $\hat{f}(i,j)$ is a distorted version of $f(i,j)$. Then, the MSE and the PSNR are, respectively, defined as:

$$MSE = \frac{1}{NM} \sum_{i=1}^{N} \sum_{j=1}^{M} \left[f(i,j) - \hat{f}(i,j) \right]^2 \tag{11}$$

and

$$PSNR = 10 \log_{10} \left(\frac{\mathcal{I}_{max}{}^2}{MSE} \right) \tag{12}$$

where $\mathcal{I}_{max}$ is the maximum possible intensity value in $f(i,j)$ ($M \times N$ size). Thus, for images of 8 bits per pixel (bpp) per single channel $\mathcal{I}_{max} = 2^8 - 1 = 255$. Thus, for 24 bpp color images the PSNR is defined in the same way that in Equation 12, while MSE is the mean of individual MSE among Red, Green, and Blue channels, so once again $\mathcal{I}_{max} = 2^8 - 1 = 255$.

An important goal of any image compression systems is to improve the correlation of the pixels, since the higher correlation at the quantization, the more efficient coding system.

We employ PSNR because is a image quality assessment extensively used in the image processing field, since this metric have favorable features, such as:

1) A convenient metrics for the purpose of optimization of image coders. For example in JPEG2000, PSNR is employed both in Optimal Rate Allocation [13], [3] and Region of interest [14], [3].
2) By definition PSNR is the difference signal between the two compared images regarding the peak or the

maximum intensity error , namely $(\mathcal{I}_{max})^2$, giving a clear meaning of the energy of overall error inside a given signal.

A. *Image* Lena

Figure 8: 512×512 Image *Lena*.

Figure 9 shows that there is an important difference between the curves obtained by JPEG2000 and JPEG2000+WQbZS. In the particular case of Image *Lena*, JPEG2000 get, on the average, 36.6397dB when the image is compressed from 0.1 to 10.5 bits per pixel. While when the proposed algorithm is introduced to JPEG2000, performance of the standardized image compression system increases its performance, on the average, 3.7284 dB, which means that modifying the quantization step of the compressor reduces the error by half approximately.

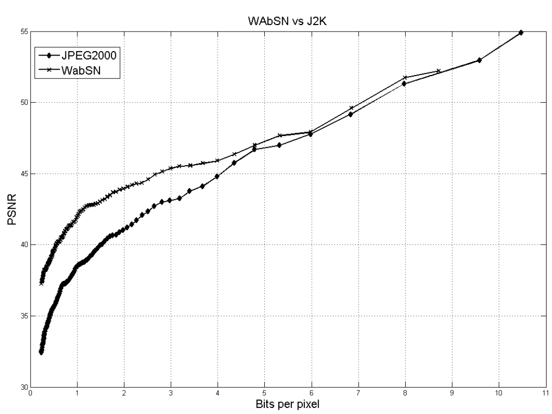


Figure 9: Bits per Pixel vs PSNR of Image *Lena*, Fig. 8.

B. CMU *Image Database*

This experiment is performed across the CMU Image Database (Annex A). Image quality estimations are assessed by PSNR.

Thus, the following experiments were performed on the selected images of *CMU* Image Database, which were transformed into YC_bC_r color representation, since it is the color space used by JPEG2000. Figure 10 shows the relation between compression rate and average quality. On average, any size image compressed by JPEG2000+WQbZS (dashed function) with 30 dB is stored at 0.65 bpp, while JPEG2000 (continuous function) stores it at 1.32 bpp.

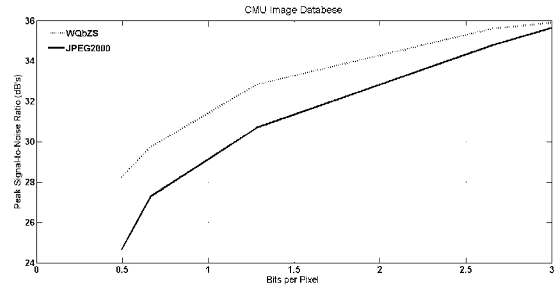


Figure 10: Comparison between JPEG2000 and JPEG2000+WQbZS image coders. Compression rate vs PSNR objective image quality, of the CMU Image database.

In Figure 11 we can see the difference when the image *Lena* is compressed at 0.5 bpp by JPEG2000 (a) and JPEG2000+WQbZS (b). At the same compression ratio, JPEG2000+WQbZS improves image quality by 1.04 dB. On average JPEG2000+WQbZS either compresses 0.27 bpp more with the same image quality or reduces in 0.93 dB the error with the same bit-rate.

C. CSIQ *Image Database*

This experiment is performed across the CSIQ Image Database (Annex B). Image quality estimations are assessed by PSNR.

Thus, the following tests are made on the selected images of *CSIQ* Image Database transformed into YC_bC_r color space (it is the color space used by JPEG2000). Figure 12 shows the relation between compression rate and average quality. On average, any size image compressed by JPEG2000+WQbZS (dashed function) with 35 dB is stored at 2.35 bpp, while JPEG2000 (continuous function) stores at 2.75 bpp.

In Figure 13 we can see the difference when the image *Bridge* is compressed at 0.35 bpp by JPEG2000 (a) and JPEG2000+WQbZS (b). At the same compression ratio, JPEG2000+WQbZS improves image quality by 1.15 dB. On average JPEG2000+WQbZS either compresses 0.28 bpp more with the same image quality or reduces in 0.947 dB the error with the same bit-rate.

VI. Conclusions

We defined Forward and Inverse statistical Quantizer using Z-score. We incorporated it to JPEG2000 proposing an alternative way dor the quantization step in the cited standard of image compression system. We expoused the mathematical explanation of normalizing or standardized the wavelet coefficients, since it increases redundancies, giving as a result a better image with the same entropy. In order to measure the effectiveness of the statistical quantization, a performance analysis is done using the image quality assessment PSNR, which measured the image quality between reconstructed and original images. Our results show that the employment of the Wavelet Quantization by means of Z-scores improves the JPEG2000 compression and image quality. In addition, when WQbZS is added into JPEG2000, it importantly improves the results getting the conventional JPEG2000 compression.

(a) JPEG2000: 0.5bpp, PSNR = 31.63 dB

(b) JPEG2000+WQbZS: 0.5bpp, PSNR = 32.67 dB

Figure 11: Comparison of bit rate of JPEG2000 and JPEG2000+WQbZS image coders, tested with the 24-bit Color Image *Lena*, taken from *CMU* Image Database.

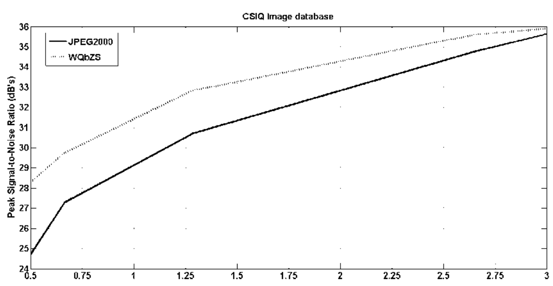


Figure 12: Comparison between JPEG2000 and JPEG2000+WQbZS image coders. Compression rate vs PSNR objective image quality, of the CSIQ Image database.

ACKNOWLEDGMENT

This work is supported by National Polytechnic Institute of Mexico (Instituto Politécnico Nacional, México) by means of Projects SIP-20160786, SIP-20161053, and SIP-20161713 the Academic Secretary and the Committee of Operation and Promotion of Academic Activities (COFAA) and National Council of Science and Technology of Mexico (CONACyT) by means of National Research System (Sistema Nacional de Investigadores) grants No. 56739 (Dr. Moreno), 32772 (Dr. Morales), and 335839 (Dr. Tejeida).

APPENDIX

A. University of Southern California Image Database

Figure 14 depicts the University of Southern California Image Data Base, *Miscellaneous volume*[15]. The database contains eight 256×256 pixel images and eight 512×512 pixel images [15].

B. Categorical Subjective Image Quality Image Database

CSIQ Database includes 30 original images (Figure 15), which are distorted by 6 different types of distortions at 4 or 5 degrees. CSIQ Database has 5000 perceptual evaluations of 25 observers[16].

REFERENCES

[1] M. Boliek, C. Christopoulos, and E. Majani, *Information Technology: JPEG2000 Image Coding System*, JPEG 2000 Part I final committee draft version 1.0 ed., ISO/IEC JTC1/SC29 WG1, JPEG 2000, April 2000.

[2] L. V. Bertalanffy, *Teoría General de los Sistemas*, M. Fondo de Cultura Ecocǿmica, Ed., 1989.

[3] D. S. Taubman and M. W. Marcellin, *JPEG2000: Image Compression Fundamentals, Standards and Practice*, ser. ISBN: 0-7923-7519-X. Kluwer Academic Publishers, 2002.

[4] M. W. Marcellin, M. A. Lepley, A. Bilgin, T. J. Flohr, T. T. Chinen, and J. H. Kasner, "An overview of quantizartion of JPEG2000," *Signal Processing: Image Communication*, vol. 17, no. 1, pp. 73–84, Jan. 2002.

[5] W. A. Pearlman and A. Said, "Image wavelet coding systems: Part II of set partition coding and image wavelet coding systems," *Foundations and Trends in Signal Processing*, vol. 2, no. 3, pp. 181–246, 2008.

[6] Z. Wang and A. C. Bovik, *Modern Image Quality Assessment*, 1st ed. Morgan & Claypool Publishers: Synthesis Lectures on Image, Video, & Multimedia Processing, February 2006.

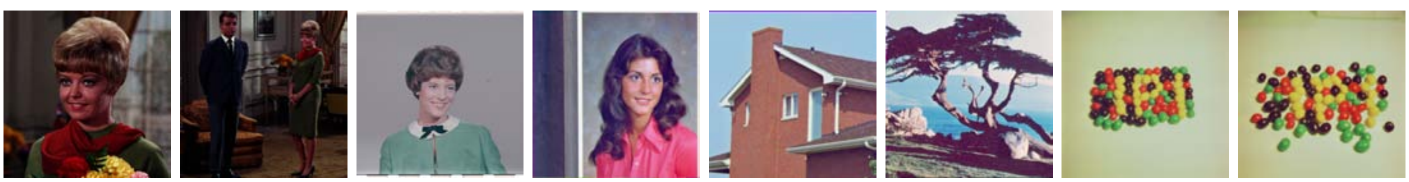
[7] A. Skodras, C. Christopoulos, and T. Ebrahimi, "The JPEG 2000 still image compression standard," *IEEE Signal Processing Magazine*, vol. 18, no. 5, pp. 36–58, September 2001.

[8] M. Antonini, M. Barlaud, P. Mathieu, and I. Daubechies, "Image coding using wavelet transform," *IEEE Transactions on Image Processing*, vol. 1, no. 2, pp. 205 – 220, April 1992.

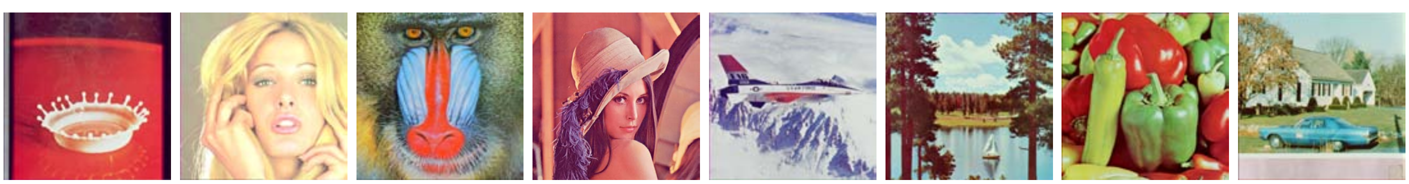
[9] W. Sweldens, "The lifting scheme: A custom-design construction of biorthogonal wavelets," *Applied and Computational Harmonic Analysis*, vol. 3, no. 2, pp. 186 – 200, 1996.

(a) JPEG2000: 0.35bpp, PSNR = 27.41 dB   (b) JPEG2000+WQbZS: 0.35bpp, PSNR = 28.56 dB

Figure 13: Comparison of bit rate of JPEG2000 and JPEG2000+WQbZS image coders, tested with the 24-bit Color Image *Bridge*, taken from *CSIQ* Image Database.



(a) Girl (b) Couple (c) Girl (d) Girl (e) House (f) Tree (g) Jelly beans (h) Jelly beans



(i) Splash (j) Tiffany (k) Baboon (l) Lena (m) F-16 (n) Sailboat (o) Peppers (p) House

Figure 14: Tested 24-bit Color Images, obtained from the University of Southern California Image Data Base. Figures (a) to (h) are 256×256 pixel Images, while Figures (i) to (p) are 512×512 pixel Images.

[10] A. Said and W. Pearlman, "A new, fast, and efficient image codec based on Set Partitioning In Hierarchical Trees," *IEEE Transactions on Circuits and Systems for Video Technology*, vol. 6, no. 3, pp. 243 –250, June 1996.

[11] E. I. Altman, "Predicting financial distress of companies: Revisiting the z-score and zeta models," *Journal of Finance*, 1968.

[12] Z. Wang and A. Bovik, "Mean squared error: Love it or leave it? a new look at signal fidelity measures," *Signal Processing Magazine, IEEE*, vol. 26, no. 1, pp. 98 –117, jan. 2009.

[13] F. Auli-Llinas and J. Serra-Sagrista, "Low complexity JPEG2000 rate control through reverse subband scanning order and coding passes concatenation," *IEEE Signal Processing Letters*, vol. 14, no. 4, pp. 251 –254, april 2007.

[14] J. Bartrina-Rapesta, F. Auli-Llinas, J. Serra-Sagrista, and J. Monteagudo-Pereira, "JPEG2000 Arbitrary ROI coding through rate-distortion optimization techniques," in *Data Compression Conference*, 25-27 2008, pp. 292 –301.

[15] S. and Image Processing Institute of the University of Southern California. (1997) The USC-SIPI image database, available at http://sipi.usc.edu/database/. Signal and Image Processing Institute of the University of Southern California. [Online]. Available: http://sipi.usc.edu/database/

[16] E. C. Larson and D. M. Chandler, "Most apparent distortion: a dual strategy for full-reference image quality assessment," in *Proc. SPIE*, vol. 742, 2009.

Figure 15: Tested 512×512 pixel 24-bit color images, belonging to the CSIQ Image database