Nonquadratic Lyapunov Functions for Nonlinear Takagi-Sugeno Discrete Time Uncertain Systems Analysis and Control

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***Abstract*—This paper deals with the analysis and design of the state feedback fuzzy controller for a class of discrete time Takagi -Sugeno (T-S) fuzzy uncertain systems. The adopted framework is based on the Lyapunov theory and uses the linear matrix inequality (LMI) formalism. The main goal is to reduce the conservatism of the stabilization conditions using some particular Lyapunov functions. Four nonquadratic Lyapunov Functions are used in this paper. These Lyapunov functions represent an extention from two Lyapunov functions existing in the literature. Their influence in the stabilization region (feasible area of stabilization) is shown through examples, the stabilization conditions of controller for discrete time T-S parametric uncertain systems is demonstrated with the variation of the lyapunov functions between (k, k+1) and (k, k+t) sample times. The controller gain can be obtained via solving several linear matrix inequalities (LMIs). Through the examples and simulations, we demonstrate their uses and their robustness. Comparative study verifies the effectiveness of the proposed methods.**

Keywords—Nonquadratic Lyapunov functions; Non-PDC; Linear Matrix Inequality; Parametric Uncertain Systems; Takagi-Sugeno

I. Introduction

Fuzzy control systems have experienced a big growth of industrial applications in the recent decades, because of their reliability and effectiveness.

In recent years, there has been growing interest in the study of stability and stabilization of Takagi–Sugeno (T–S) fuzzy system[1, 2, 3,4, 5] due to the fact that it provides a general framework to represent a nonlinear plant by using a set of local linear models which are smoothly connected through nonlinear fuzzy membership functions.

Nonlinear systems are difficult to describe. Takagi-Sugeno fuzzy model is a multimodel approach much used to modelise non linear sytems by construction with identification of input-output data [6,7]. The merit of such fuzzy model-based control methodology is that it offers an effective and exact representation of complex nonlinear systems in a compact set of state variables.With the powerful T–S fuzzy model, a natural, simple, and systematic design control approach can be provided to complement other nonlinear control techniques that require special and rather involved knowledge. Nowadays, T–S fuzzy model-based control approaches have been applied successfully in a wide range of applications.

One of the most important issues in the study of T–S fuzzy systems is the stability and stabilization analysis problems [8]. Via various approachs, a great number of stability and stabilization results for T–S fuzzy systems in both the continuous and discrete time have been reported in the literature [9,10].

Two classes of Lyapunov functions are used to analyze these systems: quadratic Lyapunov and nonquadratic Lyapunov functions. The second class of function is less conservative than the first. Many researches have investigated nonquadratic Lyapunov functions [11, 12, 13, 14, 15, 16].

Many works try to reduce the conservatism of quadratic form. Several approaches have been developed to overcome the above mentioned limitations. Piecewise quadratic Lyapunov functions were employed to enrich the set of possible Lyapunov functions used to prove stability [11]. Multiple Lyapunov functions have been paid a lot of attention due to avoiding conservatism of stability and stabilization. Some works try to enrich some properties of the membership functions [17, 18], others introduce decisions variables (slack variables) in order to provide additional degrees of freedom to the LMI problem [19, 20].

For every case, The Lyapunov function used to prove the stability has the most important effect to the results. To leave the quadratic framework, some works have dealt with nonquadratic Lyapunov functions. In this case, some results are available in the continuous and the discrete cases [21],[22],[23]. In the discrete case, new improvements has been developed in [24], by replacing the classical one sample time variation of the Lyapunov function by its variation over several samples (k samples times variations). This condition reduces the conservatism of quadratic form and give a large sets of solutions in terms of linear matrix inequality LMI. The relaxed conditions admitted more freedom in guaranteeing the stability and stabilization of the fuzzy control systems and were found to be very valuable in designing the fuzzy controller, especially when the design problem involves not only stability, but also the other performance requirements such as the speed of response, constraints on control input and output .

In this paper, a new stabilization conditions for discrete time Takagi Sugeno parametric uncertain fuzzy systems with the use of [25,26, 27] new nonquadratic Lyapunov functions are discussed. This condition was reformulated into LMI. [28,29,30,31,32,33,34,35], which can be efficiently solved by using various convex optimization algorithms.

The organization of the paper is as follows. First, T-S fuzzy modeling is discussed. Second, we discuss the proposed approachs to stabilize a T-S fuzzy system in closed loop with the new lyapunov functions. Third, simulation results show the robustness of this approachs and their influence in the stabilization region (feasible area of stabilization). We finish by a conclusion.

II. System Description and Preliminairies

In this section, we describe the concept of the Takagi-Sugeno parametric uncertain system. It's based on the state space representation.

Consider the discrete time fuzzy model T-S parametric uncertain systems for nonlinear systems given as follows.

If $z_1(t)$ is M_{i1} and… $and\ z_p(t)\ is\ M_{ip}$ then

$$\begin{cases} x(k+1)=(A_i+\Delta A_i)x(k)+(B_i+\Delta B_i)u(k) \\ y(k+1)=(C_i+\Delta C_i)x(k) \end{cases} \quad (1)$$

$$i=1......r$$

Where $M_{ij}(i=1,2....r,\ j=1,2.....p)$ is the fuzzy set and r is the number of model rules, $x(k)\in\Re^n$ is the states vector ; $u(k)\in\Re^m$ is the input vector; $A_i\in\Re^{n\times n}$,the states matrix, $B_i\in\Re^{n\times m}$ the control matrix and $z_1(k),......,z_p(k)$ are known premise variables.

The T-S fuzzy model is written under the following form:

$$\begin{cases} x(k+1)=\dfrac{\sum_{i=1}^{r}w_i(z(k))((A_i+\Delta A_i)x(k)+(B_i+\Delta B_i)u(k))}{\sum_{i=1}^{r}w_i(z(k))} \\ y(k)=\dfrac{\sum_{i=1}^{r}w_i(z(k))(C_i+\Delta C_i)x(k)}{\sum_{i=1}^{r}w_i(z(k))} \end{cases} \quad (2)$$

r : is the number of model rules.

With

$$\begin{cases} w_i(z(k))=\prod_{j=1}^{r}M_{ij}(z_j(k)) \\ h_i(z(k))=\dfrac{w_i(z(k))}{\sum_{i=1}^{r}w_i(z(k))} \qquad i=1,2,\ldots,r \end{cases} \quad (3)$$

The term $M_{ij}(z_j(k))$ is the membership degree of $z_j(k)$ in M_{ij} .

Since

$$\begin{cases} \sum_{i=1}^{r}w_i(z(k))>0 \\ w_i(z(k))\geq 0 \qquad i=1......r \end{cases} \quad (4)$$

we have

$$\begin{cases} 0<h_i(z(k))<1 \\ \sum_{i=1}^{r}h_i(z(k))=1 \end{cases} \quad (5)$$

The final output can be written under the following form

$$\begin{cases} x(k+1)=\sum_{i=1}^{r}h_i(z(k))(A_i+\Delta A_i)x(k)+(B_i+\Delta B_i)u(k) \\ \qquad =(A_{z(k)}+\Delta A_{z(k)})x(k)+(B_{z(k)}+\Delta B_{z(k)})u(k) \\ y(k)=\sum_{i=1}^{r}h_i(z(k))(C_i+\Delta C_i)x(k)=(C_{z(k)}+\Delta C_{z(k)})x(k) \end{cases} \quad (6)$$

$\Delta A_i ,\Delta B_i$ represents parametric uncertainties matrices in the state space representation. These uncertainties matrices are written under the following form.

$$\begin{cases} \Delta A=H_aFE_a \\ FF^T\leq 1 \end{cases},\begin{cases} \Delta B=H_bFE_b \\ FF^T\leq 1 \end{cases},\begin{cases} \Delta C=H_cFE_c \\ FF^T\leq 1 \end{cases} \quad (7)$$

With H_a,H_b,H_c,E_a,E_b,E_c are constants matrices.

Lemma 1, 2 and 3 present the techniques and powerful tools used through the development of the next theorems.

Lemma 1 (Schur Complement) [36,37]

Consider A,G,L,P and Q matrices with appropriates dimensions. The next properties are equivalent:

$$1.\ A^TPA-Q<0\ ,\ P>0 \quad (8)$$

$$2.\begin{bmatrix} -Q & A^TP \\ PA & -P \end{bmatrix}<0 \quad (9)$$

$$3.\exists\ G\begin{bmatrix} -Q & A^TG \\ G^TA & -G-G^T+P \end{bmatrix}<0,\ P>0 \quad (10)$$

$$4.\exists\, G,L \begin{bmatrix} -Q+A^T L^T+LA & -L+A^T G \\ -L^T+G^T A & -G-G^T+P \end{bmatrix}<0,\ P>0 \quad (11)$$

Lemma 2 [38]

Relaxaion : Whatever the choise of the Lyapunov function, the analysis of the stabilization leads us to the inequality (12) with multiple sum

$$\sum_{i_0=1}^{r}\sum_{i_{k-1}=1}^{r}\sum_{j_0=1}^{r}\sum_{j_{k-1}=1}^{r} h_{i_0}\left(z(k)\right)....h_{i_{k-1}}\left(z(2k-1)\right)h_{j_0}\left(z(k)\right)\times......$$
$$....\times h_{j_{k-1}}\left(z(2k-1)\right)\Upsilon_{i_0,...i_{k-1},j_0,...j_{k-1}}<0 \quad (12)$$

Consider $\Upsilon_{i_0,...i_{k-1},j_0,...j_{k-1}} \triangleq \tilde{\Upsilon}+\tilde{\Upsilon}^{(0)}_{i_0,j_0}+....+\tilde{\Upsilon}^{(k-1)}_{i_{k-1},j_{k-1}}$ matrices and h_i functions having the convex sum properties.

The inequality (12) is verified if the next $\left(0.5r\left(r+1\right)\right)^k$ conditions are verified

$$\forall\left(i_0,j_0\right),.....,\left(i_{k-1},j_{k-1}\right)\in\{1,2,...,r\}$$
$$\Upsilon_{i_0 i_1,...i_{k-1},j_0 j_1,...j_{k-1}}+\Upsilon_{j_0 j_1,...j_{k-1},i_0 i_1,...i_{k-1}}<0 \quad (13)$$

where

$i_0\le j_0,.....,i_{k-1}\le j_{k-1}$

Lemma 3 [39]

Consider X and $Y, Q=Q^T>0$ matrices of appropriate dimensions, the following inequality is verified

$$XY^T+YX^T\le XQX^T+YQ^{-1}Y^T \quad (14)$$

The use of these lemmas will be shown in the next section.

III. Stabilization Analysis

This section recalls the technique of the stabilization analysis of discrete T-S model based on a nonquadratic Lyapunov function. In the discrete case, we consider the variation of the Lyapunov function between two sample time. If the final equation of this variation is negative, we obtain a sufficient condition of the T-S stabilization with the state feedback controller.

Consider the discrete time fuzzy Takagi-Sugeno system under the following form.

$$\begin{cases} x(k+1)=\sum_{i=1}^{r}h_i\left(z(k)\right)\left(A_i x(k)+B_i u(k)\right) \\ \qquad =\left(A_{z(k)}x(k)+B_{z(k)}u(k)\right) \\ y(k)=\sum_{i=1}^{r}h_i\left(z(k)\right)(C_i)x(k)=C_{z(k)}x(k) \end{cases} \quad (15)$$

The non-PDC control law is described by the following equation:

$$u(k)=-\sum_{i=1}^{r}F_i G^{-1}x\left(k\right) \quad (16)$$

The Lyapunov function used in [40] expressed in equation (17)

$$V(x(k))=x^T(k)\left(\sum_{i=1}^{r}h_i\left(z\left(k\right)\right)G_i\right)^{-T}$$
$$\times\left(\sum_{i=1}^{r}h_i\left(z\left(k\right)\right)\left(P_i+\mu R\right)\right)\times\left(\sum_{i=1}^{r}h_i\left(z\left(k\right)\right)G_i\right)^{-1}x(k) \quad (17)$$
$$=x^T(k)G_{z(k)}^{-T}(P_{z(k)})G_{z(k)}{}^{-1}x(k)$$

The final equation of the Lyapunov function variation obtained by [40] which represent the stabilization condition of discrete time T-S systems is written under the following form.

$$\Upsilon_{i_0 i_1,...i_{k-1},j_0 j_1,...j_{k-1}}=$$
$$\begin{bmatrix} -P & (*) & \ddots & 0 \\ A_{i_0}G-B_{i_0}F_{j_0} & -G-G^T & \ddots & \ddots \\ \ddots & \ddots & \ddots & (*) \\ 0 & \ddots & A_{i_{k-1}}G-B_{i_{j-1}}F_{j_{k-1}} & -G-G^T+P \end{bmatrix}<0 \quad (18)$$

So [40] propose the following theorem:

Theorem [40]

Consider the discrete Takagi-Sugeno (15), the control law (16) and the $\Upsilon_{i_0 i_1,...i_{k-1},j_0 j_1,...j_{k-1}}$ defined in (18). If it exist a definite positive matrix P and matrices G, F_i , $i=\{1.....r\}$ such that the conditions (12) and (13) of lemma 2 are verified the system is globally asymptotic stable in closed loop.

We propose a new Lyapunov function based on the Lyapunov function in equation (17), by multiplying the Lyapunov matrices $P_{z(k)}$ by a scalar $\alpha>0$. So the new form of the Lyapunov function is written under the following form in equation (19).

$$V(x(k))=x^T(k)G_{z(k)}^{-T}(\alpha P_{z(k)})G_{z(k)}{}^{-1}x(k) \quad (19)$$

and the non-PDC control law is written under the following form in equation (20).

$$u(k)=-\sum_{i=1}^{r}F_i G_i^{-1}x\left(k\right) \quad (20)$$

The variation of the Lyapunov function between *k* and *k+t* sample times is given by the next equation (21)

$$\Delta_k V\left(x\left(k\right)\right)=x\left(k+t\right)^T G_{z(k)}{}^{-T}(\alpha P_{z(k)})G_{z(k)}{}^{-1}x\left(k+t\right)$$
$$-x\left(k\right)^T G_{z(k)}{}^{-T}(\alpha P_{z(k)})G_{z(k)}{}^{-1}x\left(k\right) \quad (21)$$

The final output $x\left(k+1\right)$ is written between k and (k+t) samples under the next form.

$$x\left(k+1\right)=(A_{z(k)}-B_{z(k)}F_{z(k)}G_{z(k)}{}^{-1})x\left(k\right)\ \ with\ \ A_{z(k)}=\sum_{i=1}^{r}h_i\left(z(k)\right)A_i$$
$$x\left(k+2\right)=(A_{z(k+1)}-B_{z(k+1)}F_{z(k+1)}G_{z(k)}{}^{-1})(A_{z(k)}-B_{z(k)}F_{z(k)}G_{z(k)}{}^{-1})x\left(k\right)$$

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$$x\left(k+t\right)=(A_{z(k+t-1)}-B_{z(k+t-1)}F_{z(k+t-1)}G_{z(k)}{}^{-1})\times.......\times(A_{z(k)}-B_{z(k)}F_{z(k)}G_{z(k)}{}^{-1})x\left(k\right)$$

The variation of the Lyapunov function for discrete system should be negative $\Delta_k V\left(x(k)\right)<0$. This equation is equivalent to

$$x(k)^T\begin{pmatrix}(*)G_{z(k)}^{-T}(\alpha P_{z(k)})G_{z(k)}{}^{-1}(A_{z(k+t-1)}-B_{z(k+t-1)}F_{z(k+t-1)}G_{z(k)}{}^{-1})\times....\\ ...\times(A_{z(k)}-B_{z(k)}F_{z(k)}G_{z(k)}{}^{-1})-G_{z(k)}^{-T}(\alpha P_{z(k)})G_{z(k)}{}^{-1}\end{pmatrix}x(k)<0 \quad (22)$$

The equation (22) is equivalent to equation (23)

$$\begin{pmatrix}(*)G_{z(k)}^{-T}(\alpha P_{z(k)})G_{z(k)}{}^{-1}(A_{z(k+t-1)}-B_{z(k+t-1)}F_{z(k+t-1)}G_{z(k+1)}{}^{-1})\times....\\ ...\times(A_{z(k)}-B_{z(k)}F_{z(k)}G_{z(k)}{}^{-1})-G_{z(k)}^{-T}(\alpha P_{z(k)})G_{z(k)}{}^{-1}\end{pmatrix}<0 \quad (23)$$

Consider the next modification

$$A_{z(k+t)}-B_{z(k+t)}F_{z(k+t)}G_{z(k+t)}{}^{-1}=\left(A_{z(k+t)}G_{z(k+t)}-B_{z(k+t)}F_{z(k+t)}\right)G_{z(k+t)}^{-1} \quad (24)$$

Using the congruence with the full rank matrix G, we obtain

$$(*)G_z^{-T}(\alpha P_{z(k)})\left[G_z{}^{-1}(A_{z(k+t-1)}G_{z(k+t-1)}-B_{z(k+t-1)}F_{z(k+t-1)})\right]\times.....$$
$$..\times\left[G_{z(k)}^{-1}(A_{z(k)}G_{z(k)}-B_{z(k)}F_{z(k)})\right]-(\alpha P_{z(k)})<0 \quad (25)$$

So the variation of the lyapunov function $\Delta_k V\left(x(k)\right)<0$ holds if the equation (25) is negative. The use of the Schur Complement (Lemma 1) with the equation (25) give the next equation

$$\begin{bmatrix}-\alpha P_{z(k)} & * \\ \Phi\left[G_{z(k)}^{-T}\left(A_{z(k)}G_{z(k)}-B_{z(k)}F_{z(k)}\right)\right] & \begin{pmatrix}-\Phi-\Phi^T+(*)G_{z(k)}^{-T}\alpha P_{z(k)}G_{z(k)}\times...\\ ..\times\left(A_{z(k+1)}G_{z(k+1)}-B_{z(k+1)}F_{z(k+1)}\right)\end{pmatrix}\end{bmatrix}<0 \quad (26)$$

For each iteration i={1….r}

Let's consider the following inequality:

$$\begin{bmatrix}-\alpha P_{z(k)} & * \\ \Phi\left[G_{z(k)}^{-T}\left(A_{z(k)}G_{z(k)}-B_{z(k)}F_{z(k)}\right)\right] & -\Phi-\Phi^T+\Gamma_i\end{bmatrix}<0 \quad (27)$$

with

$$\Gamma_i=(*)G_{z(k)}^{-T}\alpha P_{z(k)}G_{z(k)}\left(A_{z(k+t-1)}G_{z(k+t-1)}-B_{z(k+t-1)}F_{z(k+t-1)}\right)$$
$$\times.....\times\left(A_{z(k+i)}G-B_{z(k+1)}F_{z(k+i)}\right)$$

The application of the lemma 1 with equation (27) with $\Phi=G$ give the next inequality

$$\begin{bmatrix}-\alpha P_{i_0} & (*) & 0\\ A_{i_0}G_{j_0}-B_{i_0}F_{j_0} & -G_{i_0}-G_{i_0}^{T} & (*)\\ 0 & A_{i_{k-1}}G-B_{i_{k-1}}F_{j_{k-1}} & -G-G^T+\Gamma_2\end{bmatrix}<0 \quad (28)$$

Recursively by the use of Schur Complement we obtain the inequality (29).

$$\begin{bmatrix}-\alpha P_{z(k)} & (*) & \ddots & 0\\ A_{z(k)}G-B_{z(k)}F_{z(k)} & -G_{z(k)}-G_{z(k)}^T & \ddots & \ddots\\ \ddots & \ddots & \ddots & (*)\\ 0 & \ddots & A_{z(k+t-1)}G_{z(k+t-1)}-B_{z(k+t-1)}F_{z(k+t-1)} & -G_{z(k+t-1)}-G_{z(k+t-1)}^T+\alpha P_{z(k+t-1)}\end{bmatrix}<0$$

(29)

The use of the lemma 2 with the equation (29), give the final condition of discrete time T-S systems stabilization. This condition should be negative.

$$\Upsilon_{i_0 i_1,...i_{k-1},j_0 j_1,...j_{k-1}}=$$
$$\begin{bmatrix}-\alpha P_{i_0} & (*) & \ddots & 0\\ A_{i_0}G_{j_0}-B_{i_0}F_{j_0} & -G_{i_0}-G_{i_0}^T & \ddots & \ddots\\ \ddots & \ddots & \ddots & (*)\\ 0 & \ddots & A_{i_{k-1}}G_{j_{k-1}}-B_{i_{k-1}}F_{j_{k-1}} & -G_{i_{k-1}}-G_{i_{k-1}}^T+\alpha P_{i_{k-1}}\end{bmatrix}<0 \quad (30)$$

Therefore we state the following theorem for the discrete time Takagi-Sugeno fuzzy systems.

Theorem 1

Consider the discrete time Takagi-Sugeno (15), the control law (20) and the $\Upsilon_{i_0 i_1,...i_{k-1},j_0 j_1,...j_{k-1}}$ defined in (30). If exist a definite positive matrices P_i and matrices G_i, F_i , $i=\{1.....r\}$ and $\alpha>0$ such that the conditions (12) and (13) of lemma 2 are verified the discrete time T-S system is globally asymptotic stable in closed loop.

These two theorems represent sufficient conditions of the discrete time T-S stabilization with state feedback with k sample times variation of the Lyapunov function. In the next section, we present the analysis of the stabilization of the discrete time T-S parametric uncertain systems.

IV. Parametric Uncertain Systems Stabilization Analysis

A. *New Lyapunov Function: First approach*

In the next, we treat the case of the discrete time Takagi-Sugeno parametric uncertain systems.

Consider the uncertain system described in equation (6)

In that case, the equation (30) becomes.

$$\begin{bmatrix} -\alpha P_{z(k)} & (*) & \ddots & 0 \\ \bar{A}_{z(k)}G_{z(k)} - \bar{B}_{z(k)}F_{z(k)} & -G_{z(k)} - G_{z(k)}{}^{T} & \ddots & \ddots \\ \ddots & \ddots & \ddots & (*) \\ 0 & \ddots & \bar{A}_{z(k+t-1)}G_{z(k)} - \bar{B}_{z(k+t-1)}F_{z(k+t-1)} & -G_{z(k)} - G_{z(k)}{}^{T} + \alpha P_{z(k)} \end{bmatrix} < 0$$

$$\bar{A}_{z(k)} = A_{z(k)} + \Delta A_{z(k)} \;,\; \bar{B}_{z(k)} = B_{z(k)} + \Delta B_{z(k)} \tag{31}$$

For the uncertainties $\Delta A_{z(k)}, \Delta B_{z(k)}$, the term $\bar{A}_{z(k)}G - \bar{B}_{z(k)}F_{z(k)}$ is transformed in the following form by introducing two scalars $\tau_0 > 0,\ \mu_0 > 0$. The use of lemma 3 on uncertainties $\Delta A_{z(k)}, \Delta B_{z(k)}$ gives the next two inequalities:

$$\begin{bmatrix} 0 \\ H_a \Delta A_z \end{bmatrix} \begin{bmatrix} E_{az}G_z & 0 \end{bmatrix} + (*) \le \begin{bmatrix} \tau_0{}^{-1} G_z^T E_{az}^T E_{az} G_z & 0 \\ 0 & \tau_0 H_a \Delta A_z (\Delta A_z)^T H_a^T \end{bmatrix} \tag{32}$$

$$\begin{bmatrix} 0 \\ H_b \Delta B_z \end{bmatrix} \begin{bmatrix} E_{bz}F_z & 0 \end{bmatrix} + (*) \le \begin{bmatrix} \mu_0{}^{-1} F_z^T E_{bz}^T E_{bz} F_z & 0 \\ 0 & \mu_0 H_b \Delta B_z (\Delta B_z)^T H_b^T \end{bmatrix} \tag{33}$$

Taking in consideration the two inequality (32) and (33), the equation (31) become equation (34)

$$\begin{bmatrix} -P_{z(k)} + \Omega_0^1 & (*) & \ddots & 0 \\ A_{z(k)}G_{z(k)} - A_{z(k)}F_{z(k)} & -G_{z(k)} - G_{z(k)}^T + \Omega_0^2 & \ddots & \ddots \\ \ddots & \ddots & \ddots & (*) \\ 0 & \ddots & \Xi & \Theta \end{bmatrix} < 0 \tag{34}$$

With

$$\begin{cases} \Theta = -G_{k-1} - G_{k-1}^T + P_{k-1} + \Omega_{k-1}^2 \\ \Xi = A_{z(k+t-1)}G_{z(k+t-1)} - B_{z(k+t-1)}F_{z(k+t-1)} \\ \Omega_0^i = \tau_i{}^{-1} G_z^T E_{az(k+i)}^T E_{az(k+i)} G_z + \mu_i{}^{-1} F_z^T E_{bz(k+i)}^T E_{bz(k+i)} F_z \end{cases}$$

The use of Schur Complement (lemma1) give the next equation (35) which represents the final condition of stabilization of T-S parametric uncertain systems with the use of the Lyapunov function (19) and the control law (20).

$$\left[\begin{array}{ccccc|cccc} -\alpha P_{z(k)} & (*) & (*) & (*) & 0 & \cdots & \cdots & \cdots & 0 \\ E_{bz(k)}F_{z(k)} & -\mu_0 I & 0 & 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ E_{az(k)}G_{z(k)} & 0 & -\tau_0 I & 0 & \ddots & & & & \\ A_{z(k)}G_{z(k)} - B_{z(k)}F_{z(k)} & 0 & 0 & -G_{z(k)}^T - G_{z(k)} + \Omega_0^2 & \ddots & \ddots & & & \vdots \\ 0 & & & \ddots & \ddots & & 0 & 0 & 0 \\ \vdots & & & & \ddots & -G_{z(k)}^T - G_{z(k)} + \Omega_{k-2}^2 & (*) & (*) & (*) \\ \vdots & & & 0 & \ddots & E_{bz(k)}F_{z(k)} & -\mu_{k-1} I & 0 & 0 \\ \vdots & & & 0 & \ddots & E_{az(k)}G_{z(k)} & 0 & -\tau_{k-1} I & 0 \\ 0 & \cdots & \cdots & 0 & \ddots & A_{z(k)}G_{z(k)} - B_{z(k)}F_{z(k)} & 0 & 0 & -G_{z(k)}^T - G_{z(k)} + \Omega_{k-1}^2 + \alpha P_{z(k)} \end{array}\right] \tag{35}$$

with

$$\Omega_i^2 = \tau_i H_a H_a^T + \mu_i H_b H_b^T \tag{36}$$

After using lemma 2 the equation (35) become (37)

$$\Upsilon_{i_0 i_1,\dots i_{k-1}, j_0 j_1,\dots j_{k-1}} =$$

$$\left[\begin{array}{ccccc|cccc} -\alpha P_{i_0} & (*) & (*) & (*) & 0 & \cdots & \cdots & \cdots & 0 \\ E_{bi_0}F_{j_0} & -\mu_0 I & 0 & 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ E_{ai_0}G_{i_0} & 0 & -\tau_0 I & 0 & \ddots & & & & \\ A_{i_0}G_{j_0} - B_{i_0}F_{j_0} & 0 & 0 & -G_{i_0}^T - G_{i_0} + \Omega_0^2 & \ddots & \ddots & & & \vdots \\ 0 & & & \ddots & \ddots & & 0 & 0 & 0 \\ \vdots & & & & \ddots & -G_{i_{k-1}}^T - G_{i_{k-1}} + \Omega_{k-2}^2 & (*) & (*) & (*) \\ \vdots & & & 0 & \ddots & E_{bi_{k-1}}F_{j_{k-1}} & -\mu_{k-1} I & 0 & 0 \\ \vdots & & & 0 & \ddots & E_{ai_{k-1}}G_{j_{k-1}} & 0 & -\tau_{k-1} I & 0 \\ 0 & \cdots & \cdots & 0 & \ddots & A_{i_{k-1}}G_{j_{k-1}} - B_{i_{k-1}}F_{j_{k-1}} & 0 & 0 & -G_{k-1}^T - G_{k-1} + \Omega_{k-1}^2 + \alpha P_{i_0} \end{array}\right] < 0 \tag{37}$$

So we state the next theorem for the stabilization of the discrete time T-S parametric uncertain systems.

Theorem 2

Consider the discrete time uncertain Takagi-Sugeno system (6), the control law (20) and the $\Upsilon_{i_0 i_1,\dots i_{k-1}, j_0 j_1,\dots j_{k-1}}$ defined in (37). If exist a definite positive matrices P_i, matrices G_i, F_i, $i = \{1.....r\}$ and positives scalars τ_i, μ_i and $\alpha > 0$ such that the conditions of lemma 2 are verified the system is globally asymptotic stable in closed loop.

The next work deals with the addition of more variables in the equation $\Upsilon_{i_0 i_1,\dots i_{k-1}, j_0 j_1,\dots j_{k-1}}$ to give a large field of solutions.

In this case a condition of stabilization is developed based on new Lyapunov functions and a new non-PDC control law (20).

B. New Lyapunov Function : Second approach

Consider the new non quadratic lyapunov function in equation (38) and the non-PDC control law in equation (20). In this new function, we associate for each Lyapunov matrices $P_{z(k)}$ a

scalar α.

$$V(x(k)) = x^T(k)G_{z(k)}^{-T}(\alpha_i P_{z(k)})G_{z(k)}^{-1}x(k) \quad (38)$$

where $\alpha_i > 0$ with $i = \{1,..r\}$

Consider the same transformations and lemmas used to obtain equations (30), (37), theorems 1 and 2. The new form of equation for the stabilization of discrete time T-S fuzzy parametric uncertain systems with the use of the Lyapunov function (38) is under the following form in equation (39).

$$\Upsilon_{i_0 i_1,\ldots i_{k-1}, j_0 j_1,\ldots j_{k-1}} = \begin{bmatrix} -\alpha_{i_0}P_{i_0} & (*) & (*) & (*) & 0 & \cdots & \cdots & \cdots & 0 \\ E_{bi_0}F_{j_0} & -\mu_0 I & 0 & 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ E_{ai_0}G_{j_0} & 0 & -\tau_0 I & 0 & \ddots & & & & \\ A_{i_0}G_{j_0} - B_{i_0}F_{j_0} & 0 & 0 & -G_{j_0}^T - G_{j_0} + \Omega_0^2 & \ddots & \ddots & & & \vdots \\ 0 & & & \ddots & \ddots & & 0 & 0 & 0 \\ \vdots & & & & \ddots & -G_{j_{k-1}}^T - G_{j_{k-1}} + \Omega_{k-2}^2 & (*) & (*) & (*) \\ \vdots & & & 0 & \ddots & E_{bi_{k-1}}F_{j_{k-1}} & -\mu_{k-1}I & 0 & 0 \\ \vdots & & & 0 & \ddots & E_{ai_{k-1}}G_{j_{k-1}} & 0 & -\tau_{k-1}I & 0 \\ 0 & \cdots & \cdots & 0 & \ddots & A_{i_{k-1}}G_{j_{k-1}} - B_{i_{k-1}}F_{j_{k-1}} & 0 & 0 & -G_{j_{k-1}}^T - G_{j_{k-1}} + \Omega_{k-1}^2 + \alpha_{i_{k-1}}P_{i_{k-1}} \end{bmatrix} < 0 \quad (39)$$

With

$$\begin{cases} \Omega_{k-1}^2 = \tau_{k-1}H_a H_a^T + \mu_{k-1}H_b H_b^T \\ \Omega_{k-2}^2 = \tau_{k-2}H_a H_a^T + \mu_{k-2}H_b H_b^T \end{cases} \quad (40)$$

Using the equation (39) and the Lyapunov function (38) and the non-PDC controller (20), we propose the next theorem for the stabilization of the T-S parametric uncertain systems.

Theorem 3

Consider the discrete uncertain Takagi-Sugeno system (6), the control law (20) and the $\Upsilon_{i_0 i_1,\ldots i_{k-1}, j_0 j_1,\ldots j_{k-1}}$ defined in (39). If it exists a definite positive matrices P_i, matrices G_i, F_i, $i = \{1.....r\}$ and positives scalars τ_i, μ_i and positives scalars $\alpha_i > 0$ such that conditions of lemma 2 are verified, the system is globally asymptotic stable in the closed loop.

C. New Lyapunov Function : Third approach

The third Lyapunov function proposed in this paper represents an extention from the first Lyapunov function and the next Lyapunov function described bellow

The following Lyapunov function is used by [16,17].

$$V(x(k)) = x^T(k)\left(\sum_{i=1}^{r} h_i(z(k))G_i\right)^{-T}\left(\sum_{i=1}^{r} h_i(z(k))(P_i + \mu R)\right) \times \left(\sum_{i=1}^{r} h_i(z(k))G_i\right)^{-1} x(k) \quad (41)$$

Which P_z is symmetric and definite positive matrix, and G_z is full rank matrix. The nonlinearities are expressed by the terms $h_i(z(k)) \geq 0$ with the convex sum property $\sum_{i=1}^{r} h_i(z(k)) = 1$.

The Lyapunov function used by [13, 24], is written under the following form.

$$V(x(t)) = \sum_{k=1}^{r} h_k(z(t))V_k(x(t)) \quad (42)$$

$$V_k(x(t)) = x^T(t)(P_k + \mu R)x(t) \quad (43)$$

So the third proposed Lyapunov function is written under the following form in equation (44)

$$V(x(k)) = x^T(k)G_z^{-T}(\alpha P_z + \lambda R)G_z^{-1}x(k) \quad (44)$$

where $\alpha > 0$ and $0 \leq \lambda \leq 1$

The new form of equation for the stabilization of discrete time T-S fuzzy parametric uncertain systems with the use of the Lyapunov function (44) is under the following form in equation (45).

$$\Upsilon_{i_0 i_1,\ldots i_{k-1}, j_0 j_1,\ldots j_{k-1}, R} = \begin{bmatrix} -\alpha P_i + \mu R & (*) & (*) & (*) & 0 \\ E_{bi_0}F_{j_0} & -\mu_0 I & 0 & 0 & \ddots \\ E_{ai_0}G_{j_0} & 0 & -\tau_0 I & 0 & \ddots \\ A_{i_0}G_{j_0} - B_{i_0}F_{j_0} & 0 & 0 & -G_{j_0}^T - G_{j_0} + \Omega_0^2 & \ddots \\ 0 & & & \ddots & \ddots \\ \vdots & & & & \ddots \\ \vdots & & & 0 & \ddots \\ \vdots & & & 0 & \ddots \\ 0 & \cdots & \cdots & 0 & \ddots \end{bmatrix}$$

$$\left.\begin{matrix} \cdots & \cdots & \cdots & 0 \\ \ddots & \ddots & \ddots & \vdots \\ \ddots & & & \vdots \\ & 0 & 0 & 0 \\ -G_{j_{k-1}}^T - G_{j_{k-1}} + \Omega_{k-2}^2 & (*) & (*) & (*) \\ E_{bi_{k-1}} F_{j_{k-1}} & -\mu_{k-1} I & 0 & 0 \\ E_{ai_{k-1}} G_{j_{k-1}} & 0 & -\tau_{k-1} I & 0 \\ A_{i_{k-1}} G - B_{i_{k-1}} F_{j_{k-1}} & 0 & 0 & -G_{j_{k-1}}^T - G_{j_{k-1}} + \Omega_{k-1}^2 + \alpha P_i + \mu R \end{matrix}\right] < 0$$

(45)

With the equation (45) and the Lyapunov function (44) and the non-PDC controller (20), we propose the next theorem for the stabilization of the T-S parametric uncertain systems.

Theorem 4

Consider the discrete uncertain Takagi-Sugeno system (6), the control law (20) and the $\Upsilon_{i_0 i_1,\dots i_{k-1}, j_0 j_1,\dots j_{k-1}, R}$ defined in (45). If exist a definite positive matrices P_i, matrices R, G_i, F_i, $i = \{1.....r\}$ and positives scalars τ_i, μ_i, positive scalar $\alpha > 0$ and $0 \le \lambda \le 1$ such that the conditions of lemma 2 are verified the system is globally asymptotic stable in closed loop.

In the next section, we add more values to the LMI in order to demonstrate their influence in stabilization region by affecting to each lyapunov matrices P_i, a positive scalar α_i.

D. New Lyapunov Function : Fourth approach

The fourth Lyapunov function used in this paper is written under the following form in equation (46)

$$V(x(k)) = x^T(k) G_z^{-T} (\alpha_i P_z + \lambda R) G_z^{-1} x(k) \tag{46}$$

Under this Lyapunov function, the new condition of stabilization obtained in the next equation.

$$\Upsilon_{i_0 i_1,\dots i_{k-1}, j_0 j_1,\dots j_{k-1}, R} =$$

$$\left[\begin{matrix} -\alpha_i P_i + \mu R & (*) & (*) & (*) & 0 \\ E_{bi_0} F_{j_0} & -\mu_0 I & 0 & 0 & \ddots \\ E_{ai_0} G & 0 & -\tau_0 I & 0 & \ddots \\ A_{i_0} G - B_{i_0} F_{j_0} & 0 & 0 & -G^T - G + \Omega_0^2 & \ddots \\ 0 & & & \ddots & \ddots \\ \vdots & & & & \ddots \\ \vdots & & & 0 & \ddots \\ \vdots & & & 0 & \ddots \\ 0 & \cdots & \cdots & 0 & \ddots \end{matrix}\right.$$

$$\left.\begin{matrix} \cdots & \cdots & \cdots & 0 \\ \ddots & \ddots & \ddots & \vdots \\ \ddots & & & \vdots \\ & 0 & 0 & 0 \\ -G^T - G + \Omega_{k-2}^2 & (*) & (*) & (*) \\ E_{bi_{k-1}} F_{j_{k-1}} & -\mu_{k-1} I & 0 & 0 \\ E_{ai_{k-1}} G & 0 & -\tau_{k-1} I & 0 \\ A_{i_{k-1}} G - B_{i_{k-1}} F_{j_{k-1}} & 0 & 0 & -G^T - G + \Omega_{k-1}^2 + \alpha_i P_i + \mu R \end{matrix}\right] < 0 \tag{47}$$

We state the following theorem for the stabilization of the discrete time T-S fuzzy parametric uncertain systems.

Theorem 5

Consider the discrete uncertain Takagi-Sugeno system (6), the control law (20) and the $\Upsilon_{i_0 i_1,\dots i_{k-1}, j_0 j_1,\dots j_{k-1}, R}$ defined in (47). If it exist a definite positive set of matrices P_i, matrices R, G_i, F_i, $i = \{1.....r\}$ and positives scalars τ_i, μ_i, $\alpha_i > 0$ and $0 \le \lambda \le 1$ such that the conditions of lemma 2 are verified, then the system is globally asymptotic stable in closed loop.

Four Lyapunov functions were proposed in this paper. They represent a direct extension from two other function in the literature. In the next section, we present their robustness by showing their influence on the stabilization region.

V. Simulation and Validation of Results

Consider TS discrete uncertain system with unstable open loop models. This system is modeled with two subsystems, so we have $r = 2$.

$$A_1 = \begin{pmatrix} 0.2 & 0.27 & 0.1 \\ 0.4 & -0.6 & -0.3 \\ 0.1 & 0.5 & 0.8 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -0.3 & 0.5 & 0.1 \\ 0.2 & 0.1 & -0.9 \\ 0.1 & -0.4 & 0.7 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 1.1 \\ 0.8 \\ -0.9 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 1 \\ 0.5 \\ 0.73 \end{pmatrix} \quad H_a = \begin{pmatrix} 1 \\ 0.45 \\ 0.45 \end{pmatrix}$$

$$H_b = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad E_a = \begin{pmatrix} 0.2 & 1 & -0.4 \end{pmatrix}, \quad E_b = \begin{pmatrix} 1 \\ 0 \\ 0.19 \end{pmatrix}$$

For the simulation the membership functions are choosen as follows:

$$h_1(z(k)) = \frac{1}{1 + (0.9 x_1(k))}, \quad h_2(z(k)) = 1 - h_1(z(k))$$

With the application of theorem 2, with α=0.6 the results of LMI gives definite positive matrices P_1, P_2 and matrices G_1, G_2, F_1 and F_2:

$$P_1=\begin{pmatrix}5.4174 & 0.0220 & -0.0128\\ 0.0220 & 5.6812 & 0.0189\\ -0.0128 & 0.0189 & 5.5808\end{pmatrix},\ P_2=\begin{pmatrix}5.5756 & 0.0131 & -0.0251\\ 0.0131 & 5.8226 & -0.0135\\ -0.0251 & -0.0135 & 5.7746\end{pmatrix}$$

$$G_1=\begin{pmatrix}0.6985 & -0.0527 & 0.1130\\ -0.0160 & 0.3234 & 0.0015\\ 0.0384 & 0.0242 & 0.5142\end{pmatrix}\quad G_2=\begin{pmatrix}0.7715 & -0.0602 & 0.0654\\ -0.0224 & 0.3343 & 0.0332\\ 0.0342 & 0.0193 & 0.3660\end{pmatrix}$$

$$F1=(0.0997\ \ -0.1065\ \ -0.1300)\qquad F2=(0.0777\ \ -0.0842\ \ -0.0964)$$

With the application of theorem 4, with α=0.6 , the results of LMI gives other definite positive matrices P_1, P_2 and matrices G_1, G_2, R, F_1 and F_2 .

$$P_1=\begin{pmatrix}2.6444 & 0.0149 & -0.0295\\ 0.0149 & 2.6722 & -0.0537\\ -0.0295 & -0.0537 & 2.6953\end{pmatrix},\ P_2=\begin{pmatrix}2.6291 & -0.0149 & 0.0295\\ -0.0149 & 2.6012 & 0.0537\\ 0.0295 & 0.0537 & 2.5782\end{pmatrix},$$

$$G_1=\begin{pmatrix}0.3327 & 0.0229 & -0.0870\\ 0.0811 & 0.2159 & -0.1457\\ -0.0823 & -0.1033 & 0.2099\end{pmatrix}\ G_2=\begin{pmatrix}0.3117 & 0.0126 & -0.0188\\ 0.0093 & 0.1183 & 0.0265\\ 0.0119 & 0.0194 & 0.0491\end{pmatrix}$$

$$R=\begin{pmatrix}-4.7998 & -0.0984 & 0.2377\\ 0.1461 & -4.8765 & 0.4018\\ -0.2455 & -0.4761 & -4.9245\end{pmatrix}$$

$$F_1=(0.0399\ \ -0.0191\ \ -0.0272)\ \ ,\ \ F_2=(0.0248\ \ -0.0103\ \ -0.0176)$$

The figures 1, 2 and 3 show the convergence of state variables and the control signal to the equilibrium point zero with the application of the theorem 2.

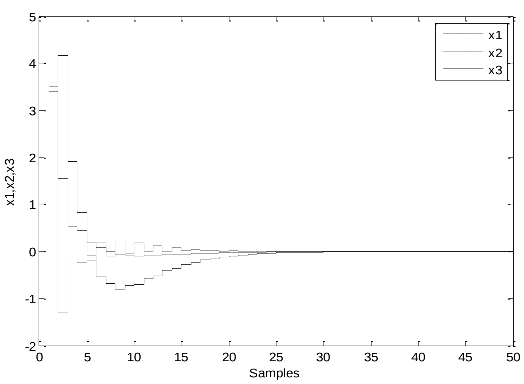

Fig. 1. Evolution of the state variables of sub-system 1

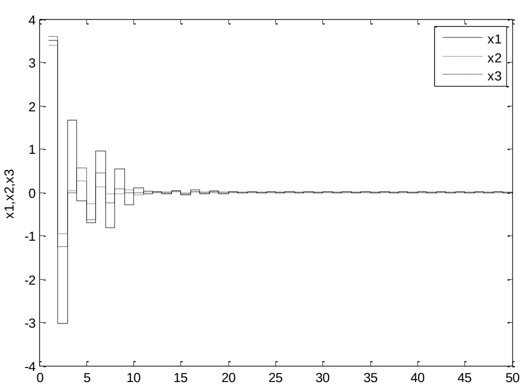

Fig. 2. Evolution of the state variables of sub-system 2

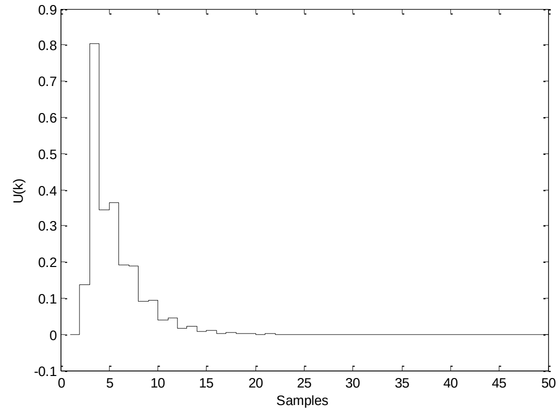

Fig. 3. Evolution of the non PDC controller signal

The next figure 4, show the feasible areas of stabilization for proposed theorems 2 and 3 and the effect of the choice of the parameters α and α_i to this areas. For the theorem 2 we choose (α=0.6) and for theorem 3 we choose

($\alpha_1 = 1,\ \alpha_2 = 1.6$)

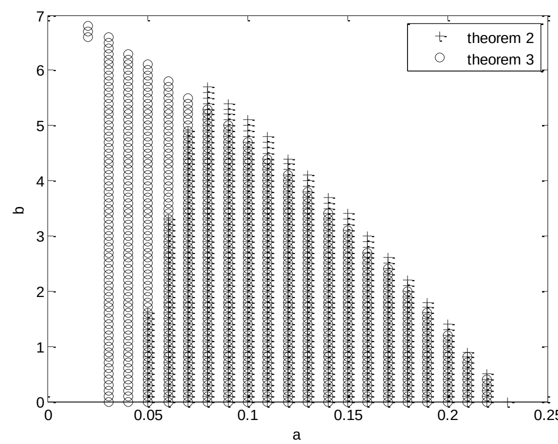

Fig. 4. Comparison between theorem 2 and 3

Theorem 3 gives a larger stabilization region than theorem 2. So by affecting for each Lyapunov matrices P a scalar α we obtain a large stabilization region than a single scalar α common to all Lyapunov matrices.

The next figures present the effect of increasing of number of parameters with $\alpha_i > 0$ in the stabilization region.

The figure 5 present the feasible area corresponding to theorem 3 with ($\alpha_1 = 0.01, \alpha_2 = 0.06$) presented by the mark (o) and ($\alpha_1 = 1,\ \alpha_2 = 1.6$) presented by the mark (+). So even we choose α_i near to zero, we obtain a larger stabilization region (feasible area of stabilization).

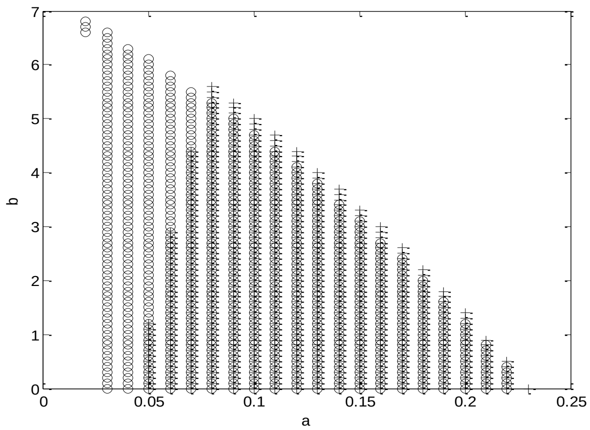

Fig. 5. Stabilization region of theorem 3

Figure 6 presents a comparison between theorem 4 and 5, it show the effect of the choice of parameters α and α_i

For the simulation, consider $(\alpha=0.6)$ for theorem 4 and $(\alpha_1=1,\ \alpha_2=1.6)$ for theorem 5. The use of theorem 5 give a large stabilization region then theorem 4.

The figure with the mark (+) represent the stabilization region of theorem 5 and the figure with the mark (o) represent the stabilization region of theorem 4.

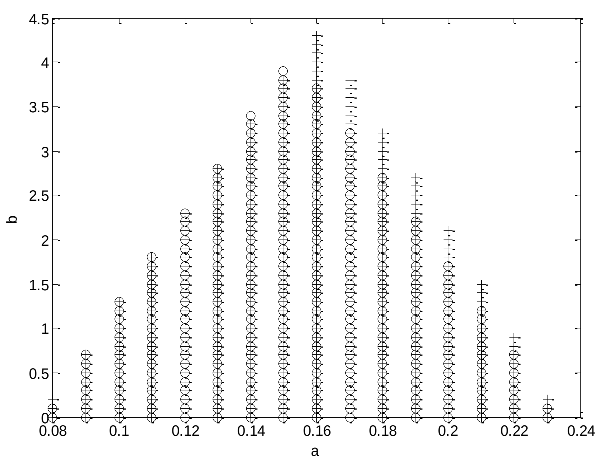

Fig. 6. Comparison between theorem 4 and 5

Figure 7 presents the effect of choice of parameter α_i near to zero with the use of theorem 5.

The figure (+) present the feasible area of stabilization for the values of ($\alpha_1=0.01,\alpha_2=0.06$) and (o) present the feasible area for the values ($\alpha_1=1,\ \alpha_2=1.6$). So to obtain a large stabilization region, α_i should be near to zero.

Figures 4,5,6 and 7 represent a comparison between the proposed theorems.

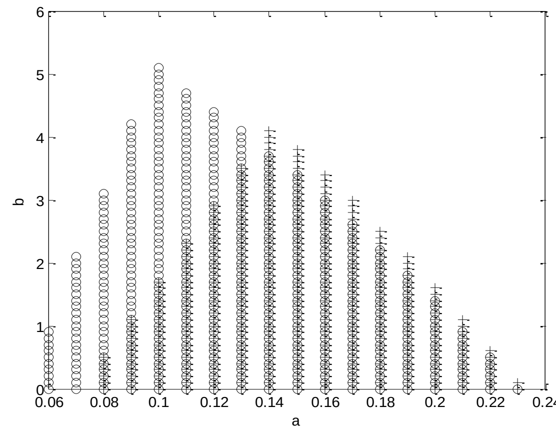

Fig. 7. Stabilization region of theorem 5

The conclusion obtained throught these figures demonstrates that the choice of a large number of parameters α_i affected to each Lyapunov matrices give a best stabilization region then one parameter and a smaller (near to zero) number also have a great effect to the stabilization region.

VI. Conclusion

This paper has developed a new fuzzy controller with state feedback for discrete time T-S parametric uncertain systems. The analysis of the stabilization problem is established by the use of Lyapunov function technique. In this case, four new Lyapunov functions are proposed. In This Lyapunov functions more parameters and slack matrix variables are introduced in order to facilitate and enrich the stabilization analysis. In the first Lyapunov function, a multiplication with a common scalar to each Lyapunov matrices is considered, In the second each Lyapunov matrices is multiplied with their own scalars. The use of the second function has a great influence to the stabilization region than the first. In the third and fourth functions, more parameters and slack matrix variables are introduced with common and single scalars for each Lyapunov matrices. Through the simulation results a single scalar for each Lyapunov matrices give a better stabilization region. The proposed theorems of stabilization was generalized between k and k+t samples time variation. Future research includes the development of design methods using these Lyapunov functions with fuzzy controller and observer in discrete time non parametric and mixed T-S uncertain systems.

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