A Robust MAI Constrained Adaptive Algorithm for Decision Feedback Equalizer for MIMO Communication Systems

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***Abstract*—Decision feedback equalizer uses prior sensor's decisions to mitigate damaging effects of intersymbol interference on the received symbols. Due to its inherent non linear nature, decision feedback equalizer outperforms the linear equalizer in case where the intersymbol interference is sever in a communication system. Equalization of multiple input multiple output fast fading channels is a daunting job as these equalizers should not only mitigate the intersymbol interference but also interstream interference. Various equalization methods have been suggested in the adaptive filtering literature for multiple input multiple output systems. In our paper, we have developed a novel algorithm for multiple input multiple output communication systems centered around constrained optimization technique. It is attained by reducing the mean squared error criteria with respect to known variance statistics of multiple access interference and white Gaussian noise. Novelty of our paper is that such a constrained method has not been used for scheme of multiple input multiple output decision feedback equalizer resulting in a constrained algorithm. Performance of the proposed algorithm is compared to the least mean squared as well as normalized least mean squared algorithms. Simulation results demonstrate that proposed algorithm outclasses competing multiple input multiple output decision feedback equalizer algorithms.**

***Index Terms*—Decision feedback equalizer, inter symbol interference, multiple access interference, Rayleigh fading, AWGN, adaptive algorithm**

I. Introduction

Multiple input multiple output (MIMO) is a diversifying technique for mobile communication systems which uses multiple antennas at the transmitting side and receiving side for an efficient channel gain capacity. Antennas installed at both side of a communication system are utilized in such a way as to reduce the errors and boost data speed resulting in an effective communication for the users. Due to its inherent performance drawbacks, single input, single output (SISO) technology, has given way to the MIMO technology. In SISO, an antenna is utilized at transmitter as well as at receiver that results in multi path phenomina. When electromagnetic field (EM) waves are blocked by barricades, mountains, tall buildings, poles etc, these waves are dispersed, and as such use numerous ways to reach the desired destination. The late arrival of disseminated waveforms (signals) results in problems such as fading, cut outs, or sporadic reception. Another drawback of SISO is observed in mobile Internet, in which it has not only reduced the data rate as but also enhanced error propagation. Utilization of MIMO technology where multiple antennas are used at both end of the channel has mitigated the multipath wave propagation problem.

Equalization of MIMO fast fading channels is a daunting task as these equalizers should not only mitigate the intersymbol interference (ISI) but also interstream interference (ISI) and this led to the introduction of decesion feedback equlaization (DFE) [1]. A DFE uses prior sensor's decisions to mitigate damaging effects of ISI on the received symbols. Because of its nonlinear behviour, DFE outperforms linear equalizer LE) in systems where ISI is sever [2].Various equalization methods have been proposed for MIMO systems [3]–[10]. In our paper, a novel algorithm for MIMO communication systems centered on constrained optimization technique is developed. It is attained by reducing the mean squared error criteria with respect to known variance statistics of the multiple access interference (MAI) and white Gaussian noise (AWGN) [11]. It is attained by reducing the mean squared error (MSE) criteria with respect to known variance statistics of MAI and AWGN.

In multiuser environment, MAI is a restrictive aspect,so a detection receiving architecture needs to be implemented which would negate the MAI and AWGN. In adaptive filtering literature, MAI is considered as part of the interfering noise, but in reality, MAI is an amorphous AWGN. In [12] MAI is detached from AWGN, but practically, it is not a valid assumption. By using the differences in the known statistical contents of the MAI and AWGN and utilizing the combination as constraint may be used to construct an algorithm that would outperform noise only constrained algorithms.

Key contributions of our paper are:

1) An MAI and AWGN constrained MMSE criterion is utilized to develop the MIMO DFE for CDMA systems and it is obtained by reducing MMSE cost function with respect to known statistics of MAI and AWGN.
2) An adaptive DFE algorithm is established which is adaptive step size. Its updating is based on variance of MAI and AWGN.
3) Statistics (variance) of MAI and AWGN is developed in a fading channel and AWGN environment.

This paper is organized as:

After introductory section, system model is explained in section II. In section III, variance of MAI and AWGN is discussed. Proposed MIMO DFE is provided in section IV. Performance of the proposed algorithm is judged by comparing it with the other algorithms in simulation result section. Finally concluding remarks are provided in section VI.

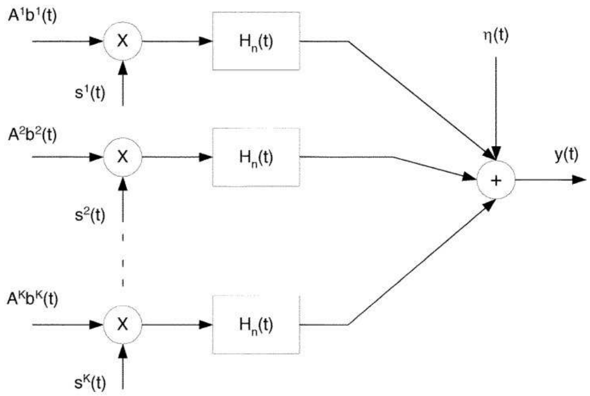

Fig. 1: Transmitter of a downlink CDMA system

II. System Model

We are using CDMA transmitter of a wireless radio setup with M transmiters and N receivers as given in Fig. 1. We are using the Rayleigh channel having impulse response between the mth transmitting antenna and nth receiving antenna of the kth signal given by

$$H_{mn}^{k}(t) = h_{mn}^{k} e^{j\theta_m}\delta(t) \tag{1}$$

where h_{mn}^{k} (($h_{mn}^{k}=1$ for AWGN) channel) is the impulse response and θ_m is phase of Rayleigh channel for kth signal. The detector in mth receiver sees the signal $r_m(t)$as given below

$$\begin{aligned}\varpi_n(t) &= \sum_{n=1}^{N}\sum_{k=-\infty}^{\infty}\sum_{i=1}^{i} C^i d_m^{k,i}\xi_m^{k,i}(t)\, h_{mn}^{k} \\ &+\varsigma_n(t), \qquad n = 1,2,3\ldots N\end{aligned} \tag{2}$$

where i is the number of subscribers, $\xi_n^{l,i}(t)$ is signature signal carrying random signature sequence of the ith subscriber given as $(k-1)T_\beta \le t \le lT_{\mathbb{C}}$. Where T_β is defined to be bit period and $T_{\mathbb{C}}$ is chip interval. Both are associated as $S_{\mathbb{C}} = T_\beta/T_{\mathbb{C}}$. $d_m^{k,i}$ is the input of the ith subscriber, C^i is the amplitude (transmitted) of the ith subscriber and $\varsigma_m(t)$ is an AWGN with zero mean and variance $\sigma^2_{\varsigma_m}$ at the nth receiving antenna. Cross correlation of signature sequences for j and l subscriber of kth signal is $\rho_k^{l,j} = \int_{(l-1)T_b}^{lT_b}\xi_m^{l}(t)\,\xi_m^{j}(t)\,dt = \sum_{i=1}^{N_c}\kappa_{k,i}^{l}\kappa_{l,i}^{j}$, where $\left\{\kappa_{k,i}^{l}\right\}$ is presumed to be normalized spreading sequence of subscriber l for the kth signal. A reason for assuming the spreading sequence to be normalized is to keep the auto correlation of the signature sequence unity.

The receiving side comprises of a bunch of matched filters, matched to signature signal of intended subscriber. We are using subscriber 1 is the intended subscriber. Output of the matched filter of the ith signal at nth receiver may be :

$$\begin{aligned} x_m^i &= \int_{(k-1)T_b}^{lT_b}\varpi_n(t)\,\xi_m^{k,i}(t) \\ &= \sum_{n=1}^{N} A^1 b_m^{k,1} h_{mn}^{k} + \sum_{m=1}^{M}\sum_{l=2}^{L} C^i b_m^{k,i}\rho_n^{i,1}(t)\, h_{mn}^{k} \\ &+\nu_n, \qquad n = 1,2,3\ldots N \end{aligned} \tag{3}$$

In this equation, 2nd componnet is MAI and is given by

$$\begin{aligned} y_n^k = \sum_{m=1}^{M}\sum_{i=2}^{i} C^i b_m^{k,i}\rho_m^{i,1}(t)\, h_{mn}^{k}, \qquad n \\ = 1,2,3\ldots N \end{aligned} \tag{4}$$

By utilizing Cauchy-Schwartz inequality, equation (4) may be set up like

$$\begin{aligned} y_n^k &\le \sum_{m=1}^{M} C^i b_m^{k,i}\rho_m^{i,1}(t)\sum_{m=1}^{M} h_{mn}^{k} \\ &\le V_n^k \sum_{i=1}^{M} h_{mn}^{k}, \qquad n = 1,2,3\ldots N \end{aligned} \tag{5}$$

where $V_n^k = \sum_{m=1}^{M}\sum_{i=2}^{L} A^l b_m^{k,l}\rho_m^{l,k}$.

III. Variance of Multiple Access Interference (MAI) in Multiple Input Multiple Output (MIMO System)

Cross-correlation $\rho^{i,1}$ is in the range $[-1,1]$ and is shown to be [13]:

$$\begin{aligned} \rho^{i,1} = \frac{(N_c - 2g)}{N_c}, \qquad g \\ = 0,1,3\ldots N_c \end{aligned} \tag{6}$$

where g is a binomial random variable carrying equal probability of failure and success. It has a mean value of $E[g] = N_c/2$ and variance is $\sigma_g^2 = N_c/4$.

As channel taps are assumed to be independent of spreading sequence as well as the data sequence, interferer's components $\sum_{m=1}^{M}\sum_{i=2}^{l} A^l b_m^{k,l}\rho_m^{l,k}$, are also not dependent on each others. When all subscribers have equal received power, variance of V_n^l can be written as

$$\begin{aligned} \sigma^2_{V_n^k} &= \sum_{m=1}^{M}\sum_{i=2}^{i} C^2 E\left[\left(d_m^{k,i}\rho_m^{i,1}\right)^2\right], \\ &= \sum_{m=1}^{M}\sum_{i=2}^{i} C^2 E\left[\left(1-\frac{2}{N_c}g\right)^2\right], \\ &= A^2\sum_{m=1}^{M}\sum_{i=2}^{i}\left(1-\frac{2}{N_c}E[g]+\frac{2}{N_c^2}E\left[g^2\right]\right), \\ \sigma^2_{V_n^k} &= \frac{C^2 N(i-1)}{N_c}, \qquad n = 1,2,3\ldots N \end{aligned} \tag{7}$$

Equation (7) is utilized in equation (5) to calculate MAI variance which may be written as

$$\begin{aligned} \sigma^2_{V_n^k} \le \frac{C^2 N(i-1)}{N_c}\sum_{m=1}^{M} E\left[\left(h_{mn}^{k}\right)^2\right], \qquad n \\ = 1,2,3\ldots N \end{aligned} \tag{8}$$

From equation (8) it is evident that impairment is sever in the MIMO system compared to the SISO system and that is due to the fact that MAI variance is dependent on the MIMO system complexity.

IV. MIMO DFE Based on Constrained Optimization Technique

ISI and MAI involved in the system shown in equation (3) can be diminished by using a MIMO DFE [14]. Our proposed equalization technique comprises of M multiple input single output (MISO) DFE in parallel (Fig. 2). The mth multiple input single output (MISO) DFE which comprised of a feedforward (FFF) and a feedback filter (FBF) having L and R taps, respectively is assigned to extract the mth stream. If the sampling window is considered to be K symbols then output of N receivers is

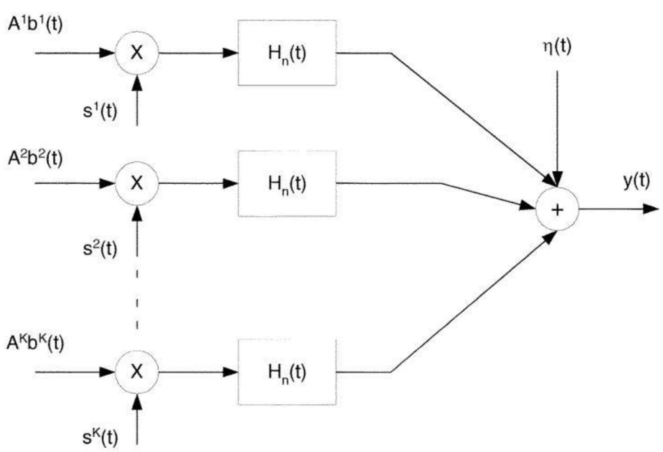

Fig. 2: MIMO DFE model

$$\mathbf{S}(k) = [s_1(k)\, s_2(k) \dots s_N(k)]^T,$$

and

$$\mathbf{S}^k = \left[\mathbf{S}^T(k)\, \mathbf{S}^T(k-1) \dots \mathbf{S}^T(k-K+1)\right]^T,$$

where $\mathbf{S}^k$ is an $ML \times 1$ vector. If $\mathbf{f}_n(l)$, for $m = 1,2,3\dots M$ represents FFF and $\mathbf{p}_n(k)$, for $n = 1,2,3\dots M$ represents FBF of length R then nth DFE output may be set below

$$\hat{u}_m^k = \mathbf{w}_m^H(k)\,\mathbf{o}^l, \qquad m = 1,2,3\dots M \tag{9}$$

where

$$\mathbf{w}_n(k) = \left[\mathbf{h}_n^T(k) \quad \mathbf{a}_n^T(k)\right]^T,$$
$$\mathbf{u}^k = \left[\mathbf{p}^{kT} \quad \check{\mathbf{b}}^{kT}\right]^T$$

and $\check{\mathbf{b}}^l$ is $MR \times 1$ vector and is a decision device's output

$$\check{\mathbf{b}}^k = \left[\check{\mathbf{b}}^{kT}(l)\, \check{\mathbf{b}}^{kT}(k-1) \dots \check{\mathbf{b}}^{kT}(k-R+1)\right]^T,$$

and

$$\check{\mathbf{b}}^l(k) = \left[\check{b}_1(k)\, \check{b}_2(k) \dots \check{b}_M(k)\right]^T.$$

FFF and FBF in equation (9) are updated by utilizing MAI and AWGN constrained LMS algorithm developed by [11] as

$$\mathbf{h}_m(k+1) = \mathbf{h}_m(k) + \alpha_m(k)\, e_m(k)\, \mathbf{y}^k \tag{10}$$
$$\mathbf{b}_m(k+1) = \mathbf{b}_m(k) + \alpha_m(k)\, e_m(k)\, \check{\mathbf{b}}^k \tag{11}$$

where $e_k(k) = \check{\mathbf{b}}^k - \hat{u}_m^k$ for $m = 1,2,3\dots M$ and $\lambda(k)$ is an adaptive learning parameter of an adaptive algorithm for MSE cost and is

$$\delta(k) = \alpha(1 + \gamma\delta(k)) \tag{12}$$
$$\lambda(k+1) = \lambda(k) + \mu\left[\frac{1}{2}\left(e^2(k) - \sigma_\alpha^2\right) - \lambda(k)\right] \tag{13}$$

where δ and μ are positive step sizes. It is evident in equation (12) that the LMS algorithm converges when $\mu = 0$. The proposed algorithm depends on the variance of MAI and AWGN shown as

$$\sigma_\alpha^2 = \sigma_{V_n^k}^2 + \sigma_{v_n^k}^2 \tag{14}$$

where $\sigma_{V_n^k}^2$is the MAI variance and$\sigma_{v_n^k}^2$ is the variance of AWGN defined earlier.

V. Discussion on Results

Simulation results are given here to judge performance of our algorithm by comparing it to LMS and normalized LMS algorithms. We have chosen a 2×2 MIMO system. Signal to noise ratio (SNR) is chosen to be twenty dB. A MIMO DFE with FFF having length five and FBF having length of two respectively are also used. Two channel settings are used while performing simulations for 4 subscribers with equal transmitted powers.

- AWGN channel
- Rayleigh channel

A. AWGN Channel Setting

In AWGN channel environment, our proposed algorithm performance is evaluated viz a viz the LMS and NCLMS algorithms using phase reversal keying and quadrature phase shift keying modulation schemes. Fig. 3 shows the performance evaluation using the phase reversal keying. As evident, our proposed MNCLMS converges faster than rest of the algorithms. It stablizes at an MSE of -18 dB in 1500 iterations. There is a bit degradation in the convergence but still has outperformed the other algorithms. A similar pattern is seen by using quadrature phase shift keying modulation scheme in Fig. 4 where there is a slight deterioration in the convergence rate but still far better then LMS and NLMS algorithms. Our proposed algorithm is converging at -18 dB in 1800 iterations

B. Rayleigh channel Setting

We are using a Rayleigh channel setting with Doppler frequency of $f_d = 250$Hz using phase reversal keying and quadrature phase shift keying modulation schemes. As seen in Fig. 5, the proposed algorithm converged faster then rest of the competing algorithms. MNCLMS algorithm achieves MSE at -19 dB in 2000 iterations. Fig. 6 depicts performance comparison of the algorithm with LMS and NCLMS algorithm for

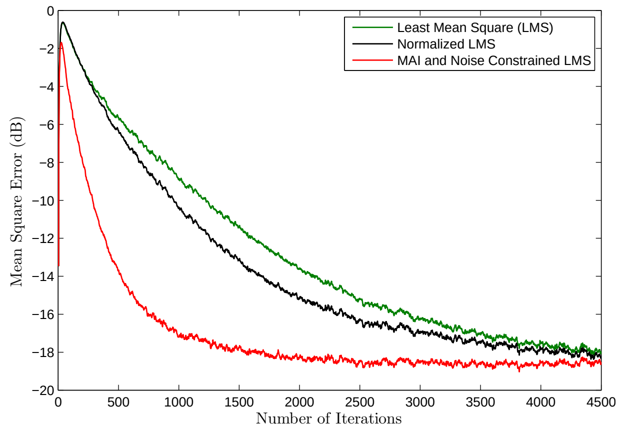

Fig. 3: Mean squared error performance comparison in an AWGN setting with 20 dB SNR and phase reversal keying modulation scheme

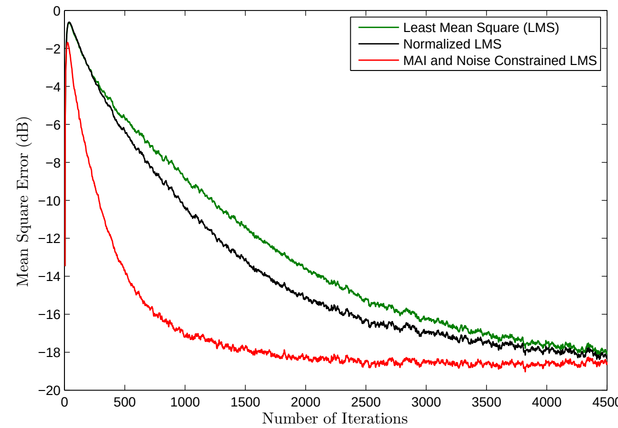

Fig. 5: Mean squared error performance comparison in Rayleigh channel environment with $f_d = 250$Hz and 20 dB SNR using phase reversal keying modulation scheme

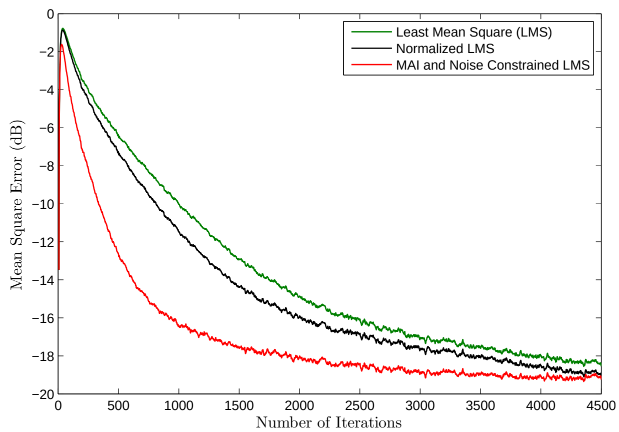

Fig. 4: Mean squared error performance comparison in AWGN setting with 20 dB SNR and quadrature phase shift keying modulation scheme

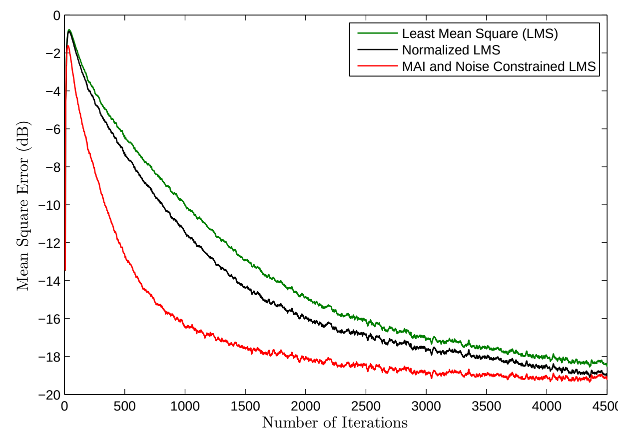

Fig. 6: Mean squared error performance comparison in Rayleigh channel setting with $f_d = 250$Hz and 20 dB SNR using quadrature phase shift keying modulation

the Rayleigh channel environment using quadrature phase shift keying modulation scheme. Our algorithm has outperformed the other competing algorithms.

VI. Conclusion

In this paper, an MAI and AWGN variance constrained algorithm for a MIMO CDMA DFE is developed. Performance of the the proposed algorithm is compared to the least mean squared as well as normalized least mean squared algorithms. Simulation results demonstrate that our algorithm has outclassed the competing algorithms.

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