

# Relative Humidity Profile Estimation Method with AIRS (Atmospheric Infrared Sounder) Data by Means of SDM (Steepest Descend Method) with the Initial Value Derived from Linear Estimation

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**Abstract**—Relative humidity profile estimation method with AIRS (Atmospheric Infrared Sounder) data by means of SDM (Steepest Descend Method) with the initial value derived from LED: Linear Estimation Method is also proposed. Through experiments, it is found that there is almost 15 (%) of relative humidity estimation error. Therefore, it can be said that the relative humidity is still tough issue for retrieval. It is also found that the estimation error does not depend on the designated atmospheric models, Mid-Latitude Summer/Winter, Tropic. Even if the assigned atmospheric model is not correct, the proposed SDM based method allows almost same estimated relative humidity. In other word, it is robust against atmospheric model.

**Keywords**—Atmospheric Infrared Sounder (AIRS); Steepest Descend Method (SDM); LED; MODTRAN; relative humidity; atmospheric model; Infrared sounder

## I. INTRODUCTION

Vertical profiles of air-temperature and relative humidity (Water vapor) can be estimated with satellite based Infrared Sounder data [1]. Retrieval accuracy, however, is not good enough for Earth system science. Estimation accuracy of air-temperature and relative humidity at tropopause<sup>1</sup> altitude is not good enough, in particular, because there are gradient changes of air-temperature and relative humidity profile in the tropopause. Therefore, observed radiance at the specific channels is not changed for the altitude.

There is least square based conventional method for estimation of vertical profiles of air-temperature and relative humidity. In the method, Root Mean Square (RMS) difference between observed radiance and calculated radiance (model based radiance) with the designated physical parameters in the model is minimized. Then, the designated physical parameters including air-temperature and relative humidity at the minimum RMS difference are to be solutions.

The most typical least square method is Newton-Raphson method<sup>2</sup> which gives one of local minima. Newton-Raphson method needs the first and the second order derivatives,

Jacobian and Hessian at around the current solution. It is not easy to formularize these derivatives analytically. The proposed method is based on Levenberg Marquardt (LM)<sup>3</sup> of non-linear least square method. It uses numerically calculated the first and the second order derivatives instead of analytical based derivatives, namely, these derivatives can be calculated with radiative transfer model based radiance calculations. At around the current solution in the solution space, directional derivatives are calculated with the radiative transfer model such as MODTRAN.

The proposed method is validated for air-temperature and relative humidity profile retrievals with Infrared: IR sounder<sup>4</sup> data obtained from AQUA/AIRS (AIRS instrument onboard AQUA satellite [2]-[7] by mean of Steepest Descent Method: SDM. Although SDM based optimization method shows relatively good accuracy, it takes a comparatively large computer resources and it is falling in local minima, not in the global optimum. To avoid this situation of which the solution is falling in local minima and accelerate the convergence through giving initial value which is derived from a Linear Estimation Method (LEM). Due to the fact that retrieval accuracy depends on the initial value, the most appropriate initial value is given from the LEM in the proposed method.

From the previous research works, a comparison of retrieving accuracy between Newton-Raphson method and the proposed method based on LM method [8] is made to demonstrate an effectiveness of the proposed method in terms of estimation accuracy for the altitude of tropopause [9]. Global Data Assimilation System: GDAS<sup>5</sup> data of assimilation model derived 1-degree mesh data is used as truth data of air-temperature and relative humidity profiles. The experimental data show that the proposed method is superior to the conventional Newton-Raphson method.

The next section describes the proposed method together with typical conventional Newton-Raphson method for retrieving vertical profiles followed by experiments. Then

<sup>1</sup> <http://en.wikipedia.org/wiki/Tropopause>

<sup>2</sup> [http://en.wikipedia.org/wiki/Newton's\\_method](http://en.wikipedia.org/wiki/Newton's_method)

<sup>3</sup> [http://en.wikipedia.org/wiki/Levenberg%E2%80%93Marquardt\\_algorithm](http://en.wikipedia.org/wiki/Levenberg%E2%80%93Marquardt_algorithm)

<sup>4</sup> [http://en.wikipedia.org/wiki/Atmospheric\\_Infrared\\_Sounder](http://en.wikipedia.org/wiki/Atmospheric_Infrared_Sounder)

<sup>5</sup> <http://www.mmm.ucar.edu/mm5/mm5v3/data/gdas.html>

conclusion with some discussions is followed by together with future research works.

## II. PROPOSED METHOD

### A. Infrared Sounder

Fig. 1 shows atmospheric transmittance and transmittance of carbon dioxide and water vapor. Using absorption bands of molecules, carbon dioxide and water vapor content in the atmosphere can be retrieved. There is strong relation between air-temperature and carbon dioxide content. Meanwhile, water vapor content is a closely related to relative humidity. Therefore, air-temperature and relative humidity can be estimated using absorption characteristics of carbon dioxide and water vapor contents in the atmosphere.

Using slope characteristics of the absorption characteristics, vertical profile of carbon dioxide and water vapor can also be estimated. Optical depths<sup>6</sup> of molecules are different and depend on the wavelength at the slope of the absorption characteristics. First derivative of optical depth against altitude is called weighting function.<sup>7</sup> Therefore, weighting function is a function of altitude.

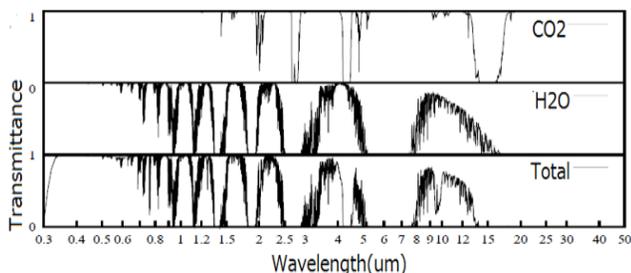


Fig. 1. Atmospheric transmittance and transmittance of carbon dioxide and relative humidity.

Fig. 2(a) shows an example of a transmittance as a function of altitude while (b) shows an example of a weighting function.

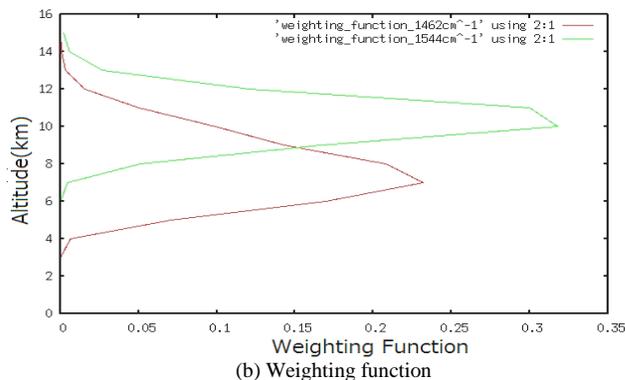
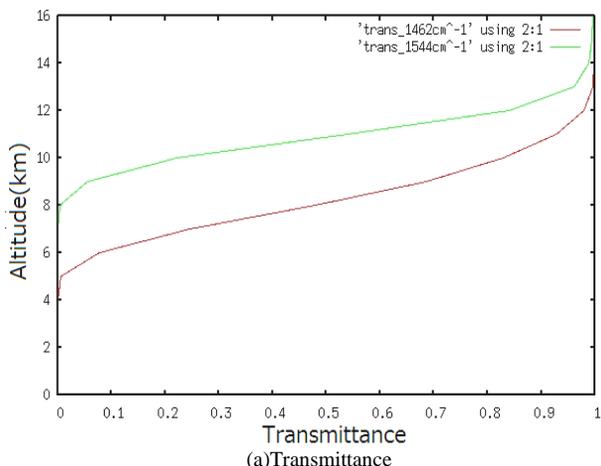


Fig. 2. Transmittance as a function of altitude and weighting function.

Mission instruments which allow vertical profile retrievals in the infrared wavelength region onboard satellite are called as infrared sounder. There are some infrared sounders such as TIROS-N / NOAA TOVS- High Resolution Infrared Sounder model 2 (HIRS/2)<sup>8</sup>. One of infrared sounders which are widely used with high quality of mission instruments is AIRS<sup>9</sup>: Atmospheric Infrared Sounder. Although AIRS data quality is high enough, retrieval accuracy is not good enough. Estimation accuracy of air-temperature and relative humidity at around tropopause is not good enough.

### B. Conventional Retrieval Method

The previously proposed method for air-temperature and relative humidity retrievals is intended to improve retrieval accuracy. Typically, the conventional retrieval method is minimizing formulated error covariance using Newton-Raphson method. It, however, gives one of local minima sometime. Although the simulated annealing method gives global optimum solution, it takes huge computational resources.

The proposed method is based on non-linear least square method of Levenberg Marquardt: LM. LM method minimizes the difference between AIRS data derived radiance,  $R_{0i}$  and estimated radiance,  $R_i$  based on atmospheric model with the parameters of air-temperature and relative humidity.

$$S = \sum_{i=0}^n (R_i - R_{0i})^2 \quad (1)$$

Thus, the geophysical parameter of air temperature and water vapor (relative humidity) at the difference of radiance reaches to the minimum is estimated. Widely used, reliable and accurate enough atmospheric software code of MODTRAN<sup>10</sup> which allows calculate at sensor radiance with a variety of parameters is used in the proposed method. Solution update equation of Newton-Raphson method is expressed by (2).

$$X_{n+1} = X_n - H^{-1}J(X_n) \quad (2)$$

<sup>6</sup> [http://en.wikipedia.org/wiki/Optical\\_depth](http://en.wikipedia.org/wiki/Optical_depth)

<sup>7</sup> [http://www.ecmwf.int/newsevents/training/rcourse\\_notes/DATA\\_ASSIMILATION/INVERSION\\_METHODS/inversion\\_methods2.html](http://www.ecmwf.int/newsevents/training/rcourse_notes/DATA_ASSIMILATION/INVERSION_METHODS/inversion_methods2.html)

<sup>8</sup> [www.ozonelayer.noaa.gov/action/tovs.htm](http://www.ozonelayer.noaa.gov/action/tovs.htm)

<sup>9</sup> [http://aqua.nasa.gov/about/instrument\\_airs.php](http://aqua.nasa.gov/about/instrument_airs.php)

<sup>10</sup> <http://en.wikipedia.org/wiki/MODTRAN>

where  $H$  denotes Hessian matrix<sup>11</sup> which consists of the second order derivatives (residual square error,  $S$  which is represented by (1) by geophysical parameter, air-temperature and water vapor (relative humidity)). Also,  $J$  denotes Jacobean<sup>12</sup> which consists of the first derivative of vectors ( $S$  by geophysical parameters of air temperature and water vapor (relative humidity),  $x$ ). On the other hand, solution update equation is expressed by (3).

$$X_{n+1} = X_n + (J^T J + I)^{-1} J^T (R_i - R_{0i}) \quad (3)$$

The first derivative is represented by (4) while the second order derivative is expressed by (5), respectively.

$$\frac{\partial S}{\partial x_i} = -2 \sum_{k=1}^n (R_k - R_{0k}) \frac{\partial R_{0k}}{\partial x_i} \quad (4)$$

$$\frac{\partial^2 S}{\partial x_i \partial x_j} = 2 \sum_{k=1}^n \left[ \frac{\partial R_{0k}}{\partial x_i} \frac{\partial R_{0k}}{\partial x_j} - (R_k - R_{0k}) \frac{\partial^2 R_{0k}}{\partial x_i \partial x_j} \right] \quad (5)$$

On the other hand, the first and second order derivatives of  $R$  with  $x$  are expressed by (6) and (7), respectively.

$$\frac{\partial R_{0k}}{\partial x_i} \leftarrow \text{MODTRAN} \quad (6)$$

$$\frac{\partial^2 R_{0k}}{\partial x_i \partial x_j} \leftarrow \frac{\partial R_{0k}}{\partial x_i} * \frac{\partial R_{0k}}{\partial x_j} \quad (7)$$

Equation (6) can be derived from MODTRAN. Thus, the geophysical parameter, air temperature and water vapor relative humidity at the difference of radiance reaches to the minimum is estimated.

In the previously proposed method, these derivatives are calculated numerically. In order to determine the derivatives, 2% changes of relative humidity are considered for calculation of derivative  $R$  and  $S$  while 0.5K changes of air-temperature is also considered. Thus, the geophysical parameter of air temperature and water vapor (relative humidity) at when the difference of radiance reaches to the minimum is estimated.

### C. Proposed Retrieval Method

Radiative transfer equation is expressed as follows:

$$Rv = (I_0)_v \tau_v(z_0) + \int_{z_0}^{\infty} Bv \{T(z)\} K_v(z) dz \quad (8)$$

where  $R, I, \tau, z, B, T, K$  denotes the top of the atmosphere radiance (at sensor radiance), extinction component from the atmosphere, transparency of the atmosphere, altitude, brightness temperature of the ground surface, surface temperature, extinction function, respectively. This can be simplified and rewritten as follows:

$$R = BK \quad (9)$$

Then,  $R$  can be minimized as (10) through changing atmospheric transparency which is closely related to relative humidity content in the atmosphere which results in relative humidity profile retrieval.

$$R - R_0 = \frac{\partial R}{\partial q} (q - q_0) \quad (10)$$

In this process,  $R$  matrix of radiative brightness temperature is square matrix. Therefore, it can be solved relatively easily.

The estimated matrix can be expressed as follows:

$$\hat{x} = x_a + (A^T S_\varepsilon^{-1} A + S_a^{-1})^{-1} A^T S_\varepsilon^{-1} (R - R_a) \quad (11)$$

where  $x_a, A, S, R_a$  denotes designated matrix, Jacobean matrix which is expressed with (12), measuring error covariance matrix which is expressed with (13), and designated radiative brightness temperature.

$$A = \begin{pmatrix} \frac{\partial R_1}{\partial q_1} & \frac{\partial R_1}{\partial q_2} & \frac{\partial R_1}{\partial q_n} \\ \frac{\partial R_2}{\partial q_1} & \frac{\partial R_2}{\partial q_2} & \frac{\partial R_2}{\partial q_n} \\ \frac{\partial R_n}{\partial q_1} & \frac{\partial R_n}{\partial q_2} & \frac{\partial R_n}{\partial q_n} \end{pmatrix} \quad (12)$$

$$S_{ij} = \varepsilon (x_i - \hat{x}_i)(x_j - \hat{x}_j)^T \quad (13)$$

Then, optimum solution can be obtained by Steepest Descent: Method: SDM method as follows:

$$q_k = q_{k-1} + \alpha_k g_k \quad (14)$$

where  $\alpha, g$  denotes step size and direction of the next solution, respectively. Process flow of the SDM method can be represented in Fig. 3.

Initial value is very important. If the initial value is close to the global optimum solution, SDM allows to reach the global optimum solution easily while the initial value is far from the global optimum solution and is close to one of a plenty of local minima, then SDM reaches a local minima easily. In the proposed method, the initial value is given by Lear Estimation Method: LEM. Generally, it is considered the LEM gives a solution which is situated at an appropriate solution space not far from the global optimum solution. Therefore, it is expected that the proposed method allows much appropriate solution rather than the SDM with arbitrary initial value not from the LEM in terms of residual error and computational resources.

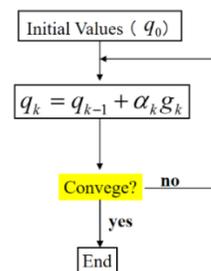


Fig. 3. Process flow of the SDM method.

<sup>11</sup> [http://en.wikipedia.org/wiki/Hessian\\_matrix](http://en.wikipedia.org/wiki/Hessian_matrix)

<sup>12</sup> [http://en.wikipedia.org/wiki/Jacobian\\_matrix\\_and\\_determinant](http://en.wikipedia.org/wiki/Jacobian_matrix_and_determinant)

### III. EXPERIMENT

#### A. Preliminary Experiment

AIRS data of the Mexican gulf at latitude of 20 degree North and longitude of 49-degree West which is acquired at 18:00 UTC on November 16, 2002 is used for the experiment. The intensive study area is shown in Fig. 4.

Also, Mid-Latitude Winter model of MODTRAN is used with the standard relative humidity, ozone and Carbone

dioxide together with the standard water vapor and air-temperature profiles because the Mexican gulf is situated in the Mid-Latitude region.

From the AIRS data, 9 channels of data are selected as shown in Table I. The top of the atmosphere brightness temperature (K) in the wave number range of 1460-1620  $\text{cm}^{-1}$  is shown in Fig. 5.



Fig. 4. Intensive study area.

TABLE I. SELECTED AIRS CHANNELS TOGETHER WITH THEIR WAVE NUMBER

WAVE NUMBER	1478	1483	1508	1514	1519	1541	1544	1558	1585
AIRS CHANNEL	1684	1692	1731	1740	1748	1761	1765	1786	1824

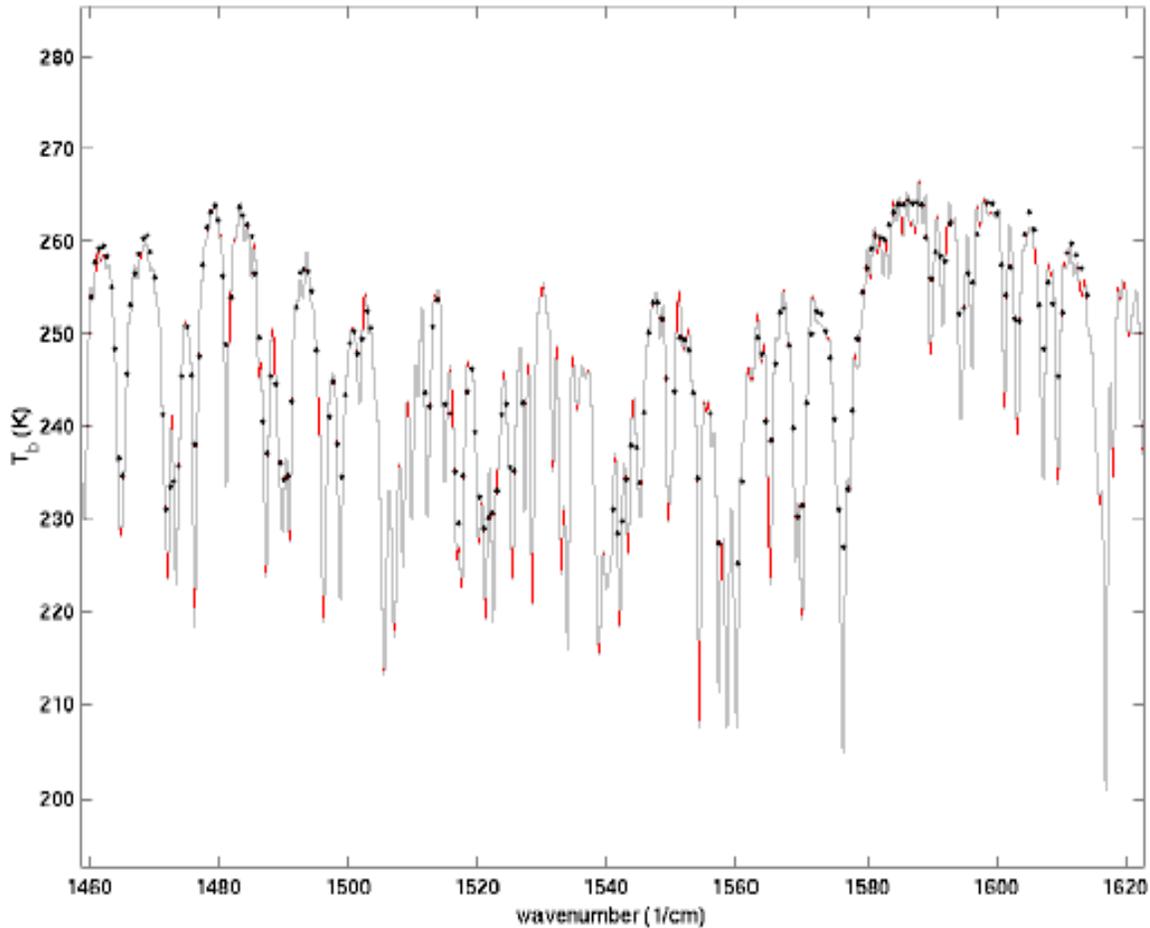


Fig. 5. The top of the atmosphere brightness temperature (K) in the wave number range of 1460-1620  $\text{cm}^{-1}$ .

The peaks of the weighting functions imply that the most sensitive altitudes to the relative humidity profile.

By using these brightness temperature and weighting functions, relative humidity profile can be estimated based on the proposed method with the SDM method. The procedure of the proposed relative humidity profile estimation is shown in Fig. 6. Meanwhile, weighting functions of these channels are shown in Fig. 7.

The most sensitive wave number of AIRS channels are selected. Then the top of the atmosphere brightness temperature is calculated with MODTRAN. Starting with the initial point  $RH_0$  which is estimated by using the method based on linear model of problem solving, the best matched brightness temperature is to be found by changing relative humidity profile.

First, wave number together with the corresponding weighting function is selected. Then, the top of the atmosphere brightness temperature is calculated using MODTRAN. The best relative humidity profile is estimated by minimizing the difference between observed brightness

temperature and the MODTRAN derived brightness temperature using the SDM method.

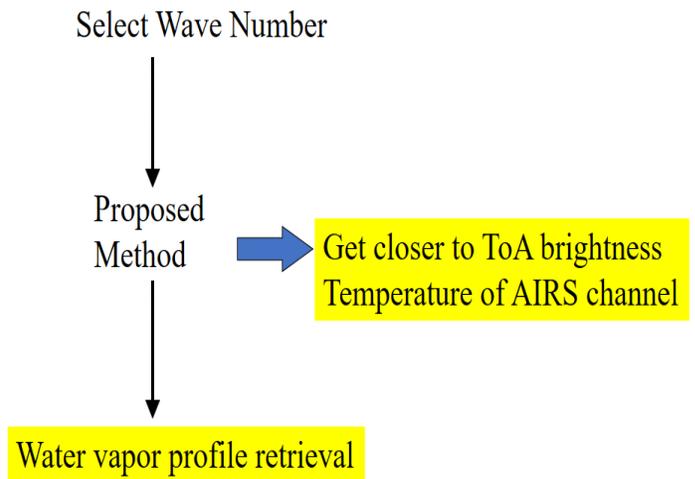


Fig. 6. Procedure of the proposed relative humidity profile estimation method.

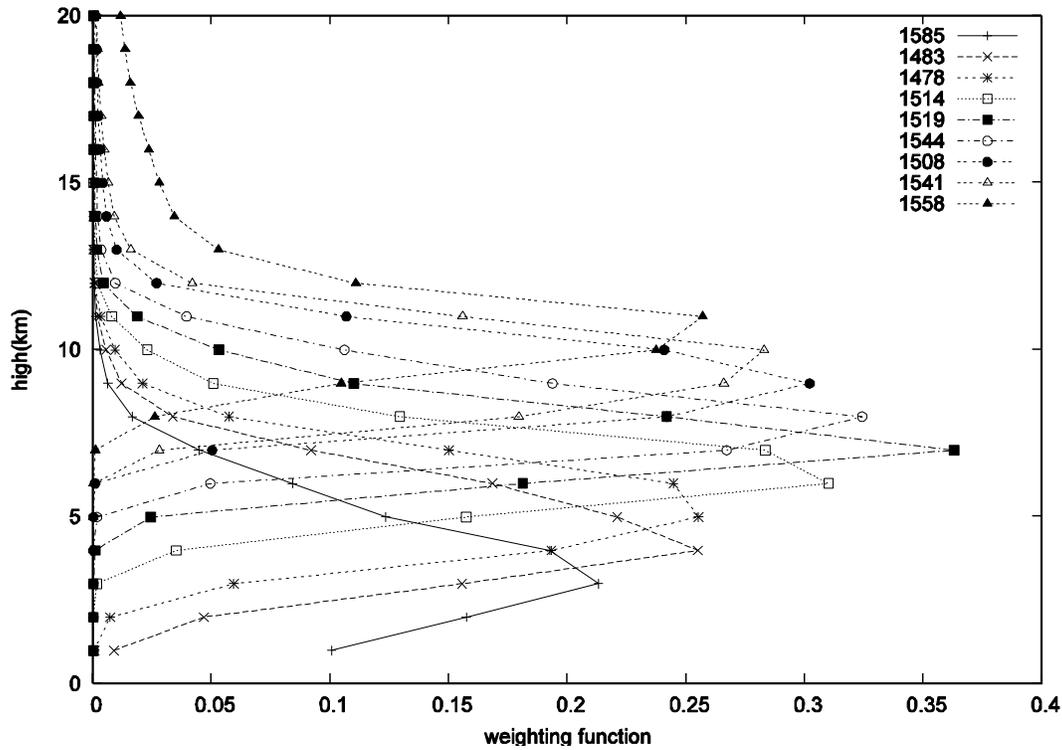


Fig. 7. Weighting functions of the selected 9 wave number channels

**B. Retrieved Result**

The retrieved result of relative humidity profile based on the proposed method is shown in Fig. 8. Meanwhile, the difference between calculated and acquired brightness temperature is shown in Fig. 9. There are three retrieved results in the Fig. 8, Mid-Latitude Winter, Mid-Latitude Summer and Tropic atmosphere models of MODTRAN. The AIRS data used is acquired in November 2002. Therefore, Mid-Latitude Winter would be better for relative humidity

profile estimation. It, however, is not true in some cases as shown in Fig. 8. Trend of the retrieved relative humidity profile with Mid-Latitude Winter coincide to the MODTRAN derived profile.

Meanwhile, the difference between calculated and estimated brightness temperature shows different trends depending on the wave number. It is not always the difference decreases in accordance with increasing of altitude.

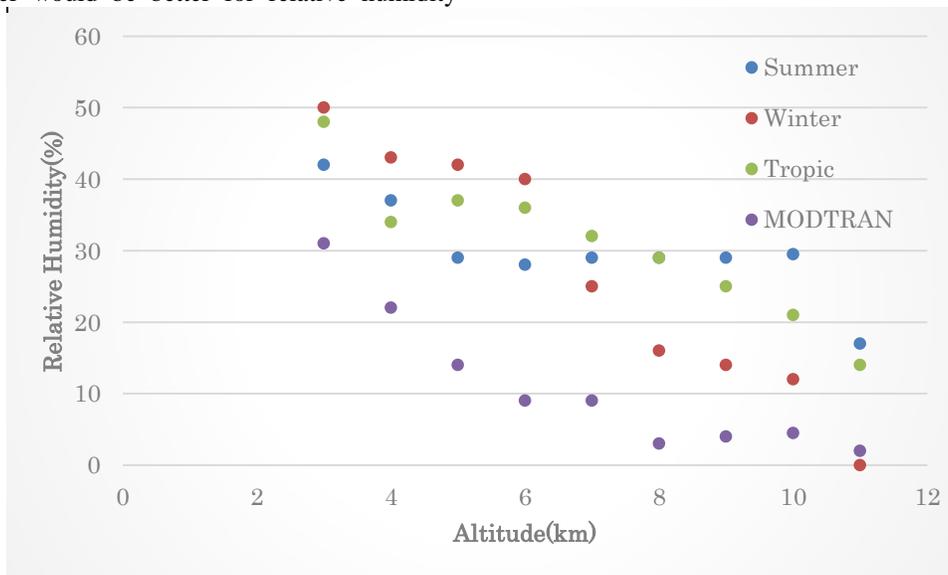


Fig. 8. Estimated relative humidity profile.

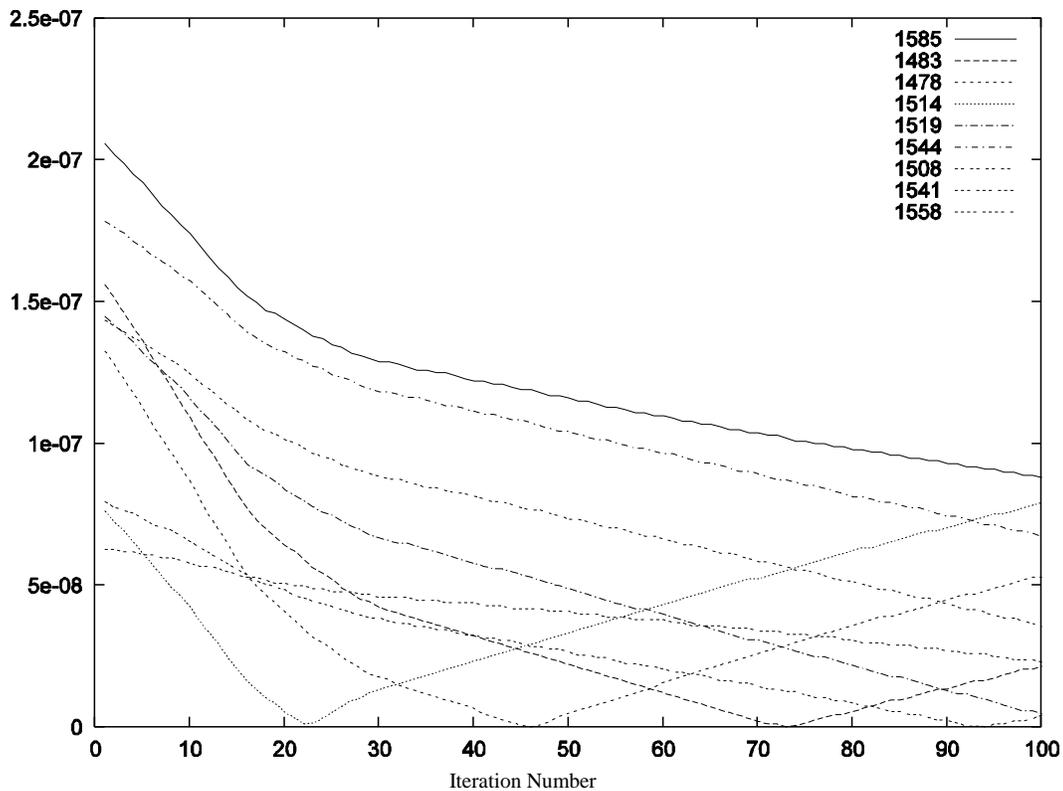


Fig. 9. Difference between calculated and acquired brightness temperature (K).

It is found that there is almost 15 (%) of relative humidity estimation error. Therefore, it can be said that the relative humidity is still tough issue for retrieval. It is also found that the estimation error does not depend on the designated atmospheric models, Mid-Latitude Summer/Winter, Tropic. Even if the assigned atmospheric model is not correct, the proposed SDM based method allows almost same estimated relative humidity. In other word, it is robust against atmospheric model.

## VI. CONCLUSION

Non-linear optimization method based on Levenberg-Marquardt of non-linear least square method with numerically calculate deviation of Jacobean and Hessian is proposed for relative humidity and air-temperature profile estimations with infrared sounder data. Through comparisons with the conventional Newton-Raphson method, it is confirmed that the proposed method is superior to the conventional Newton-Raphson method in terms of retrieving accuracy. Also, the proposed method does not need Jacobean and Hessian because first and second order derivatives are calculated numerically with MODTRAN code based radiative transfer model.

It is found that there is almost 15 (%) of relative humidity estimation error. Therefore, it can be said that the relative humidity is still tough issue for retrieval. It is also found that the estimation error does not depend on the designated atmospheric models, Mid-Latitude Summer/Winter, Tropic. Even if the assigned atmospheric model is not correct, the proposed SDM based method allows almost same estimated

relative humidity. In other word, it is robust against atmospheric model.

Further investigation is required for improvement of relative humidity retrieval accuracy.

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